DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 8

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(c)**

We know that the function

$\phi(x) = (x-a)^2 \sin\left(\frac{1}{x-a}\right)$

Is continuous and differentiable at $x = a$ whereas the function $\Psi(x) = |x-a|$ is everywhere continuous but not differentiable at $x = a$

Therefore, $f(x)$ is not differentiable at $x = 1$

2 **(d)**

$\lim\limits_{x\to 0} \frac{2^x - 2^{-x}}{x} = \lim\limits_{x\to 0} 2^x \log 2 + 2^{-x} \log 2$

[by L' Hospital's rule]

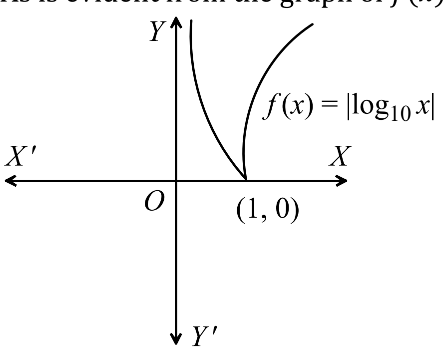
$= \log 4$

Since, the function is continuous at $x = 0$

$\therefore \quad f(0) = \lim\limits_{x\to 0} f(x) \Rightarrow \quad f(0) = \log 4$

3 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$f''(1^+) = \lim\limits_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$

$\Rightarrow f''(1^+) = \lim\limits_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$\Rightarrow f''(1^+) = \lim\limits_{h\to 0} \frac{\log_{10}(1+h) - 0}{h}$

$$\Rightarrow f''(1^+) = \lim_{h\to 0}\frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$$

$$f''(1^-) = \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0}\frac{f(1-h)-f(1)}{h}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0}\frac{\log_{10}(1-h)}{h} = \lim_{h\to 0}\frac{\log_e(1-h)}{h\log_e 10} = -\log_{10} e$$

4 **(a)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h} \qquad [\because f(x+y) = f(x)+f(y)]$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(h)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\sin h\ g(h)}{h} = \lim_{h\to 0}\frac{\sin h}{h}\lim_{h\to 0} g(h) = g(0) = k$$

5 **(a)**

We have,

$$f(x) = |x| + |x-1| = \begin{cases} -2x+1, & x<0 \\ 1, & 0\le x<1 \\ 2x-1, & 1\le x \end{cases}$$

Clearly, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 1 = 1$, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2x-1) = 1$

and, $f(1) = 2\times 1 - 1 = 1$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

So, $f(x)$ is continuous at $x = 1$

Now, $\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{1-1}{-h} = 0$

and,

$$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{2(1+h)-1-1}{h} = 2$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$

6 **(d)**

The given function is differentiable at all points except possibly at $x = 0$

Now,

(RHD at $x = 0$)

$$= \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{h+1}-1}{h^{3/2}}$$

$$= \lim_{h\to 0} \frac{h}{h^{3/2}(\sqrt{h+1}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{h}(\sqrt{h+1}+1)} \to \infty$$

So, the function is not differentiable at $x = 0$

Hence, the required set is $R - \{0\}$

7 **(a)**

We have,

$f(x)\, f(y) = f(x) + f(y) + f(xy) - 2$

$\Rightarrow f(x).\, f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) + f(1) - 2$

$\Rightarrow f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ $\left[\because f(1) = 2 \text{ (Putting } x = y = 1 \text{ in the given relation)}\right]$

$\Rightarrow f(x) = x^n + 1$

$\Rightarrow f(2) = 2^n + 1$

$\Rightarrow 5 = 2^n + 1$ $[\because f(2) = 5 \text{ (given)}]$

$\Rightarrow n = 2$

$\therefore f(x) = x^2 + 1 \Rightarrow f(3) = 10$

8 **(b)**

We have,

$f(x) = \frac{1}{2}x - 1$, for $0 \le x \le \pi$

$$\therefore \{f(x)\} = \begin{cases} -1, \text{ for } 0 \le x < 2 \\ 0, \text{ for } 2 \le x \le \pi \end{cases}$$

$$\Rightarrow \tan[f(x)] = \begin{cases} \tan(-1) = -\tan(1), & 0 \le x < 2 \\ \tan 0 = 0, & 2 \le x \le \pi \end{cases}$$

It is evident from the definition of $\tan[f(x)]$ that

$\lim_{x\to 2^-} \tan[f(x)] = -\tan 1$ and, $\lim_{x\to 2^+} \tan[f(x)] = 0$

So, $\tan[f(x)]$ is not continuous at $x = 2$

Now,

$$f(x) = \frac{1}{2}x - 1 \Rightarrow f(x) = \frac{x-2}{2} \Rightarrow \frac{1}{f(x)} = \frac{2}{x-2}$$

Clearly, $f(x)$ is not continuous at $x = 2$

So, $\tan[f(x)]$ and $\tan\left[\frac{1}{f(x)}\right]$ both are discontinuous at $x = 2$

9 **(c)**

$$\lim_{x\to 0}(1+x)^{\cot x} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{x \cot x}$$

$$= \lim_{x\to 0} e^{x \cot x} = e$$

Since $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) = \lim_{x\to 0} f(x) = e$

10 **(b)**

$\text{LHL} = \lim_{h\to 0} f\left(\frac{\pi}{4} - h\right)$

$$= \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4} - h\right) - \cot\left(\frac{\pi}{4} - h\right)}{\frac{\pi}{4} - h - \frac{\pi}{4}}$$

$$= \lim_{h\to 0} \frac{-\sec^2\left(\frac{\pi}{4} - h\right) - \operatorname{cosec}^2\left(\frac{\pi}{4} - h\right)}{-1} = 4$$

[by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$, then LHL $= f\left(\frac{\pi}{4}\right)$

$\therefore \quad a = 4$

11 **(a)**

If $-1 \le x < 0$, then

$$f(x) = \int_{-1}^{x} |t|\, dt = \int_{-1}^{x} -t\, dt = -\frac{1}{2}(x^2 - 1)$$

If $x \ge 0$, then

$$f(x) = \int_{-1}^{0} -t\, dt + \int_{-1}^{x} -t\, dt = \frac{1}{2}(x^2 + 1)$$

$$\therefore f(x) = \begin{cases} -\frac{1}{2}(x^2 - 2), & -1 \le x < 0 \\ \frac{1}{2}(x^2 + 1), & 0 \le x \end{cases}$$

It can be easily seen that $f(x)$ is continuous at $x = 0$

So, it is continuous for all $x > -1$

Also, $Rf'(0) = 0 = Lf'(0)$

So, $f(x)$ is differentiable at $x = 0$

$$\therefore f'(x) = \begin{cases} -x, & -1 < x = 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$$

Clearly, $f'(x)$ is continuous at $x = 0$

Consequently, it is continuous for all $x > -1$ i.e. for $x + 1 > 0$

Hence, f and f' are continuous for $x + 1 > 0$

12 **(c)**

We have,

$$f(x) = \lim_{n\to\infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}$$

$$\Rightarrow f(x) = \lim_{n\to\infty} \frac{1 - x^{2n}}{1 + x^{2n}}$$

$$\Rightarrow f(x)=\begin{cases}\frac{1-0}{1+0}=1, & \text{if} -1<x<1\\ \frac{1-1}{1+1}=0, & \text{if } x=\pm 1\\ \frac{0-1}{0+1}=-1, & \text{if } |x|>1\end{cases}$$

Clearly, $f(x)$ is discontinuous at $x=\pm 1$

13 **(b)**

Clearly, $\log|x|$ is discontinuous at $x=0$

$f(x)=\frac{1}{\log|x|}$ is not defined at $x=\pm 1$

Hence, $f(x)$ is discontinuous at $x=0, 1, -1$

14 **(a)**

For continuity, $\lim_{x\to 0} f(x)=k$

$\Rightarrow \lim_{x\to 0} \frac{\sin 3x}{\sin x}=k \Rightarrow \lim_{x\to 0} \frac{\sin 3x}{3x}.\frac{3x}{\sin 3x}=k$

$\Rightarrow 3=k$

15 **(b)**

Since, the function $f(x)$ is continuous

$\therefore \quad f(0)=\text{RHL } f(x)=\text{LHL} f(x)$

Now, RHL $f(X)=\lim_{h\to 0}\frac{\log(1+0+h)+\log(1-0-h)}{0+h}$

$=\lim_{h\to 0}\frac{\log(1+h)+\log(1-h)}{h}$

$=\lim_{h\to 0}\frac{\frac{1}{1+h}-\frac{1}{1-h}}{1}=0$

[by L 'Hospital's rule]

$\therefore \quad f(0)=\text{RHL } f(x)=0$

16 **(d)**

$$f(x)=\begin{cases}\frac{x-4}{|x-4|}+a, & x<4\\ a+b, & x=4\\ \frac{x-4}{|x-4|}+b, & x>4\end{cases}=\begin{cases}-1+a, x<4\\ a+b\\ 1+b, x>4\end{cases}$$

$\text{LHL}=\lim_{x\to 4^{\mp}} f(x)=a-1$

$\text{RHL}=\lim_{x\to 4^{\mp}} f(x)=1+b$

Since, LHL=RHL $=f(4)$

$\Rightarrow a-1=a+b=b+1$

$a=1$ and $b=-1$

17 **(d)**

We have,

$$f(x)=\begin{cases}\dfrac{-1}{x-1}, & 0<x<1\\ \dfrac{1-1}{x-1}=0, & 1<x<2\\ 0, & x=1\end{cases}$$

Clearly, $\lim_{x\to1^-} f(x)\to -\infty$ and $\lim_{x\to1^+} f(x)=0$

So, $f(x)$ is not continuous at $x=1$ and hence it is not differentiable at $x=1$

18 **(d)**

$$\lim_{x\to\frac{\pi}{4}} f(x) = \lim_{x\to\frac{\pi}{4}}\frac{1-\sqrt{2}\sin x}{\pi-4x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}\cos x}{4}=\frac{1}{4}$$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x=\frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right) \Rightarrow \frac{1}{4}=a$$

19 **(d)**

$$\text{LHL}=\lim_{x\to1^-} f(x)=\lim_{h\to0} 1-h+a=1+a$$

$$\text{RHL}=\lim_{x\to1^+} f(x)=\lim_{h\to0} 3-(1+h)^2=2$$

For $f(x)$ to be continuous, LHL=RHL

$$\Rightarrow 1+a=2 \Rightarrow a=1$$

20 **(b)**

$$\text{LHL}=\lim_{h\to0}\frac{\cos 3(0-h)-\cos(0-h)}{(0-h)^2}$$

$$=\lim_{h\to0}\frac{\cos 3h-\cos h}{h^2}$$

$$=\lim_{h\to0}\frac{-3\sin 3h+\sin h}{2h}$$

$$=\lim_{h\to0}\frac{-9\cos 3h+\cos h}{2}=\frac{-9+1}{2}=-4$$

$$\because \lim_{x\to0^-} f(x)=f(0) \Rightarrow \lambda=-4$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	A	A	D	A	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	A	B	D	D	D	D	B