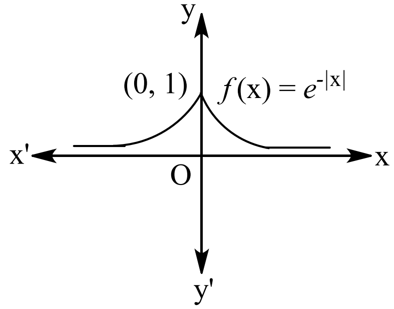
DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth SUBJECT : MATHS
DATE : DPP NO. : 7

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(a)**

It is clear from the figure that $f(x)$ is continuous everywhere and not differentiable at $x = 0$ due to sharp edge



2 **(c)**

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}} \times \frac{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}}{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}} \times \frac{\sqrt{a + x} + \sqrt{a - x}}{\sqrt{a + x} + \sqrt{a - x}}$$

$$\Rightarrow \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{-2ax(\sqrt{a + x} + \sqrt{a - x})}{2x\,(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2})}$$

$$= \frac{-a(2\sqrt{a})}{(a + a)} = -\sqrt{a}$$

3 **(b)**

Given, $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k & x = 0 \end{cases}$

$$\text{LHL} = \lim_{x \to 0^-} f(x)$$

$$= \lim_{h \to 0} \frac{1 - \cos 4(0 - h)}{8(0 - h)^2}$$

$$= \lim_{h \to 0} \frac{1 - \sin 4h}{8h^2}$$

$$= \lim_{h \to 0} \frac{4 \sin 4h}{16h} = 1 \text{ [by L 'Hospital's rule]}$$

Since, $f(x)$ is continuous at $x=0$

$\therefore \quad f(0)=\text{LHL} \Rightarrow \quad k=1$

4 **(d)**

Given, $f(x)=|x-1|+|x-2|+\cos x$

Since, $|x-1|, |x-2|$ and $\cos x$ are continuous in [0, 4]

$\therefore$ $f(x)$ being sum of continuous functions is also continuous

5 **(c)**

If function $f(x)$ is continuous at $x=0$, then

$f(0)=\lim_{x\to 0} f(x)$

$\therefore \quad f(0)=k=\lim_{x\to 0} x\sin\frac{1}{x}$

$\Rightarrow k=0 \qquad \left[\because -1\le \sin\frac{1}{x}\le 1\right]$

6 **(b)**

We have,

$h(x)=\{f(x)\}^2+\{g(x)\}^2$

$\Rightarrow h'(x)=2f(x)2f'(x)+2\,g(x)\,g'(x)$

Now,

$f'(x)=g(x)$ and $f''(x)=-f(x)$

$\Rightarrow f''(x)=g'(x)$ and $f''(x)=-f(x)$

$\Rightarrow -f(x)=g'(x)$

Thus, we have

$f'(x)=g(x)$ and $g'(x)=-f(x)$

$\therefore h'(x)=-2\,g(x)g'(x)+2\,g\,(x)g'(x)=0$, for all x

$\Rightarrow h(x)=$ Constant for all x

But, $h(5)=11$. Hence, $h(x)=11$ for all x

7 **(a)**

$$f(x)=|x|^3=\begin{cases} 0, & x=0 \\ x^3, & x>0 \\ -x^3, & x<0 \end{cases}$$

Now, $Rf'(0)=\lim_{h\to 0}\frac{h^3-0}{h}=0$

And $Lf'(0)=\lim_{h\to 0}\frac{-h^3-0}{-h}=0$

$\because \quad Rf'(0)=Lf'(0)=0$

$\therefore \quad f'(0)=0$

8 **(b)**

We have,

$(\text{LHL at } x=0)=\lim_{n\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$\Rightarrow (\text{LHL at } x=0)=\lim_{n\to 0}\sin^{-1}(\cos\,(-h)) = \lim_{h\to 0}\sin^{-1}(\cosh h)$

$\Rightarrow (\text{LHL at } x=0)=\sin^{-1}1=\pi/2$

$(\text{RHL at } x=0)=\lim_{x\to 0^+} f(x)$

$\Rightarrow$(RHL at $x=0$) $= \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \sin^{-1}(\cos h)$

$\Rightarrow$(RHL at $x=0$) $= \sin^{-1}(1) = \pi/2$

and, $f(0) = \sin^{-1}(\cos 0) = \sin^{-1}(1) = \pi/2$

$\therefore$ (LHL at $x=0$) = (RHL at $x=0$) = $f(0)$

So, $f(x)$ is continuous at $x=0$

Now,

$$f'(x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}} = \frac{\sin x}{|\sin x|} = \begin{cases} \dfrac{-\sin x}{-\sin x} = 1, x < 0 \\ \dfrac{-\sin x}{\sin x} = -1, x > 0 \end{cases}$$

$\therefore$ (LHD at $x=0$) $= 1$ and (RHD at $x=0$) $= -1$

Hence, $f(x)$ is not differentiable at $x=0$

9 **(d)**

For any $x \neq 1, 2$, we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore, $f(x)$ is continuous for all $x \neq 1, 2$

Continuity at $x=1$:

We have,

$$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$$

$$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{h\to 0} \frac{(1-h-2)(1-h+2)(1-h+1)(1-h-1)}{|(1-h-1)(1-h-2)|}$$

$$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{h\to 0} \frac{(3-h)(2-h)(-1-h)(-h)}{|(-h)(-1-h)|}$$

$$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{h\to 0} \frac{(3-h)(2-h)h(h+1)}{h(h+1)} = 6$$

and,

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$$

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} \frac{(1+h-2)(1+h+2)(1+h+1)(1+h-1)}{|(1+h-1)(1+h-2)|}$$

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} \frac{(h-1)(3+h)(2+h)(h)}{|h(h-1)|}$$

$$\lim_{x\to 1^+} f(x) = -\lim_{h\to 0} \frac{(h-1)(3+h)(2+h)h}{h(1-h)} = -6$$

$$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

So, $f(x)$ is not continuous at $x=1$

Similarly, $f(x)$ is not continuous at $x=2$

10 **(b)**

Let $f(x) = \frac{g(x)}{h(x)} = \frac{x}{1+|x|}$

It is clear that $g(x) = x$ and $h(x) = 1+|x|$ are differentiable on $(-\infty, \infty)$ and $(-\infty, 0) \cup (0, \infty)$ respectively

Thus, $f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.Now, we have to check the differentiable at $x=0$

$$\therefore \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\frac{x}{1+|x|}-0}{x}=\lim_{x\to 0}\frac{1}{1+|x|}=1$$

Hence, $f(x)$ is differntaible on $(-\infty,\infty)$

11 **(b)**

At $x=0$,

$$\text{LHL}=\lim_{h\to 0}\frac{1}{1-e^{-1/(0-h)}}=\lim_{h\to 0}\frac{1}{1-e^{1/h}}=0$$

$$\text{RHL}=\lim_{h\to 0}\frac{1}{1-e^{-1/(0+h)}}=\lim_{h\to 0}\frac{1}{1-e^{-1/h}}=1$$

$\therefore$ FUnction is not continuous at $x=0$

12 **(a)**

We have,

$fog=I$

$\Rightarrow fog(x)=x$ for all x

$\Rightarrow f'(g(x))g'(x)=1$ for all x

$$\Rightarrow f'(g(a))=\frac{1}{g'(a)}=\frac{1}{2}\Rightarrow f'(b)=\frac{1}{2}\quad [\because f(a)=b]$$

13 **(a)**

Since, $\lim_{x\to 0} f(x)=f(0)$

$$\Rightarrow \lim_{x\to 0}\frac{\sin \pi x}{5x}=k$$

$$\Rightarrow (1)\frac{\pi}{5}=k \Rightarrow k=\frac{\pi}{5}\quad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

14 **(d)**

Given, $f(x)=[x], x\in(-3.5, 100)$

As we know greatest integer is discontinuous on integer values.

In given interval, the integer values are

$(-3,-2,-1,0,\ldots,99)$

$\therefore$ Total numbers of integers are 103.

15 **(a)**

$$\text{LHL}=\lim_{h\to 0}f(0-h)$$

$$=\lim_{h\to 0}\frac{e^{-1/h}-1}{e^{-1/h}+1}=-1\quad\left[\because \lim_{h\to 0}\frac{1}{e^{1/h}}=0\right]$$

$$\text{RHL}=\lim_{h\to 0}f(0+h)=\lim_{h\to 0}\frac{e^{1/h}-1}{e^{1/h}+1}$$

$$=\lim_{h\to 0}\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}}=1$$

$\therefore$ LHL $\neq$ RHL

So, limit does not exist at $x=0$

16 **(d)**

We have,

$$f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)} + \dots$$

$$\Rightarrow f(x) = \frac{x^4}{1 - \frac{1}{1+x^4}} = 1 + x^4, \text{ if } x \neq 0$$

Clearly, $f(x) = 0$ at $x = 0$

Thus, we have

$$f(x) = \begin{cases} 1 + x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Clearly, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is neither continuous nor differentiable at $x = 0$

17 **(a)**

We have,

$$f(x) = \begin{cases} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{cases}$$

$\therefore g(x) = fof(x)$

$\Rightarrow f(x) = f(f(x))$

$$\Rightarrow g(x) = \begin{cases} f(1 + x), & 0 \leq x \leq 2 \\ f(3 - x), & 2 < x \leq 3 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} 1 + (1 + x), & 0 \leq x \leq 1 \\ 3 - (1 + x), & 1 < x \leq 2 \\ 1 + (3 - x), & 2 < x \leq 3 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} 2 + x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \\ 4 - x, & 2 < x \leq 3 \end{cases}$$

Clearly, $g(x)$ is continuous in $(0, 1) \cup (1, 2) \cup (2, 3)$ except possibly at $x = 0, 1, 2$ and 3

We observe that

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (2 + x) = 2 = g(0)$

and $\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} 4 - x = 1 = g(3)$

Therefore, $g(x)$ is right continuous at $x = 0$ and left continuous at $x = 3$

At $x = 1$, we have

$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} 2 + x = 3$

and, $\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} 2 - x = 1$

$\therefore \lim_{x \to 1^+} g(x) \neq \lim_{x \to 1^-} g(x)$

So, $g(x)$ is not continuous at $x = 1$

At $x = 2$, we have

$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (2 - x) = 0$

and,

$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (4 - x) = 0$

$\therefore \lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$

So, $g(x)$ is not continuous at $x = 2$

Hence, the set of points of discontinuity of $g(x)$ is $\{1, 2\}$

18 **(b)**

Since $g(x)$ is the inverse of function $f(x)$

$\therefore gof(x) = I(x)$, for all x

Now, $gof(x) = I(x)$, for all x

$\Rightarrow gof(x) = x$, for all x

$\Rightarrow (gof)'(x) = 1$, for all x

$\Rightarrow g'(f(x))f'(x) = 1$, for all x [Using Chain Rule]

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$, for all x

$\Rightarrow g'(f(c)) = \frac{1}{f'(c)}$ [Putting $x = c$]

19 **(d)**

Given, $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore it is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x\to 0} x^p \cos\left(\frac{1}{x}\right) = 0 \Rightarrow p > 0$

As $f(x)$ is differentiable at $x = 0$

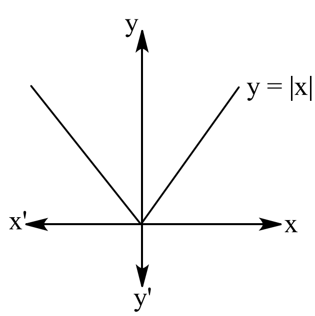
$\therefore \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$ exists finitely

$\Rightarrow \lim_{x\to 0} \frac{x^p \cos\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x\to 0} x^{p-1} \cos\frac{1}{x} - 0$ exists finitely

$\Rightarrow \quad p - 1 > 0 \quad \Rightarrow \quad p > 1$

20 **(a)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except at $x = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	D	C	B	A	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	A	D	A	D	A	B	D	A