DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

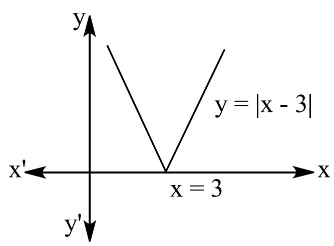
SOLUTIONS

SUBJECT : MATHS
DPP NO. : 6

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(a)**

From the graph it is clear that $f(x)$ is continuous everywhere but not differentiable at $x = 3$



2 **(b)**

Given, $f(x) = \begin{cases} \frac{2x-3}{2x-3}, & \text{if } x > \frac{3}{2} \\ \frac{-(2x-3)}{2x-3}, & \text{if } x < \frac{3}{2} \end{cases}$

$= \begin{cases} 1, & \text{if } \ x > \dfrac{3}{2} \\ -1, & \text{if } x < \dfrac{3}{2} \end{cases}$

Now, RHL $= \lim\limits_{x \to \frac{3^+}{2}} f(x) = \lim\limits_{x \to \frac{3^+}{2}} 1 = 1$

And LHL $= \lim\limits_{x \to \frac{3^-}{2}} f(x) = \lim\limits_{x \to \frac{3^-}{2}} (-1) = -1$

$\because$ RHL $\neq$ LHL

$\therefore$ $f(x)$ is discontinuous at $x = \frac{3}{2}$

3 **(c)**

Since the functions $(\log t)^2$ and $\frac{\sin t}{t}$ are not defined on $(-1, 2)$. Therefore, the functions in options (a) and (b) are not defined on $(-1, 2)$

The function $g(t) = \frac{1-t+t^2}{1+t+t^2}$ is continuous on $(-1, 2)$ and

$f(x) = \int_0^x \frac{1-t+t^2}{1+t+t^2} dt$ is the integral function of $g(t)$

Therefore, $f(x)$ is differentiable on $(-1, 2)$ such that $f'(x) = g(x)$

4 **(c)**

Since, $f(x) = \frac{1-\tan x}{4x-\pi}$

Now, $\lim\limits_{x \to \pi/4} f(x) = \lim\limits_{x \to \pi/4} \left(\frac{1-\tan x}{4x-\pi}\right)$

$$= \lim_{x\to\pi/4}\left(\frac{-\sec^2 x}{4}\right) = -\frac{1}{2}$$

Since, $f(x)$ is continuous at

$x = \frac{\pi}{4}$

$$\therefore \lim_{x\to\pi/4} f(x) = f\left(\frac{\pi}{4}\right) = -\frac{1}{2}$$

5 **(a)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{1-\cos x}{x} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}.x = 0$$

Also, $f(0) = k$

For, $\lim_{x\to 0} f(x) = f(0) \Rightarrow k = 0$

6 **(a)**

We have,

$f(x) = |x| + |x-1|$

$$\Rightarrow f(x) = \begin{cases} -2x+1, & x<0 \\ x-x+1, & 0\le x<1 \\ x+x-1, & x\ge 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} -2x+1, & x<0 \\ 1, & 0\le x<1 \\ 2x-1, & x\ge 1 \end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x) = 1 = \lim_{x\to 0^+} f(x)$ and $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

So, $f(x)$ is continuous at $x = 0, 1$

7 **(d)**

$$f(0) = \lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$$

$$= \lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}$$

$$= \frac{2-1}{2+1} = \frac{1}{3}$$

9 **(b)**

$$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1+h-1}{2(1+h)^2-7(1+h)+5}-\left(\frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left(\frac{1}{2h-3}+\frac{1}{3}\right)}{h} = \lim_{h\to 0}\left(\frac{2h}{3h(2h-3)}\right) = -\frac{2}{9}$$

10 **(a)**

$\text{LHL} = \lim_{h\to 0} f\left(-\frac{\pi}{2} - h\right) = \lim_{h\to 0} 2\cos\left(-\frac{\pi}{2} - h\right) = 0$

$\text{RHL} = \lim_{h\to 0} f\left(-\frac{\pi}{2} + h\right) = \lim_{h\to 0} 2\ a\sin\left(-\frac{\pi}{2} + h\right) + b$

$= -a + b$

Since, function is continuous.

$\therefore$ RHL=LHL $\Rightarrow$ $a = b$

From the given options only (a) *ie*, $\left(\frac{1}{2}, \frac{1}{2}\right)$ satisfies this condition

11 **(a)**

We have,

$f'(0) = 3$

$\Rightarrow \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = 3$

$\Rightarrow \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = 3$ [Using:(RHD at $x = 0$) = 3]

$\Rightarrow \lim_{h\to 0} \frac{f(0)f(h) - f(0)}{h} = 3$ $\left[\begin{matrix} \because f(x+y) = f(x)f(y) \\ \therefore f(0+h) = f(0)f(h) \end{matrix}\right]$

$\Rightarrow f(0)\left(\lim_{h\to 0} \frac{f(h)-1}{h}\right) = 3$ ...(i)

Now, $f(x+y) = f(x)f(y)$ for all $x, y \in R$

$\Rightarrow f(0) = f(0)f(0)$

$\Rightarrow f(0)\{1 - f(0)\} = 0 \Rightarrow f(0) = 1$

Putting $f(0) = 1$ in (i), we get

$\lim_{h\to 0} \frac{f(h)-1}{h} = 3$ ...(ii)

Now,

$f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$

$\Rightarrow f'(5) = \lim_{h\to 0} \frac{f(5)f(h) - f(5)}{h}$

$\Rightarrow f'(5) = \left\{\lim_{h\to 0} \frac{f(h)-1}{h}\right\} f(5) = 3 \times 2 = 6$ [Using (ii)]

12 **(c)**

We have,

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f(x)' = \lim_{h\to 0} \frac{f(x) + f(h) - f(x)}{h}$

$\Rightarrow f(x)' = \lim_{h\to 0} \frac{f(h)}{h}$

$\Rightarrow f(x)' = \lim_{h\to 0} \frac{h\ g\ (h)}{h} \lim_{h\to 0} g(h) = g(0)$ [$\because g$ is conti. at $x = 0$]

13 **(b)**

The domain of $f(x)$ is $[2, \infty)$

We have,

$$f(x)=\sqrt{\frac{(\sqrt{2x-4})^2}{2}+2+2\sqrt{2x-4}}$$
$$+\sqrt{\frac{(\sqrt{2x-4})^2}{2}+2-2\sqrt{2x-4}}$$
$$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\sqrt{(\sqrt{2x-4})^2+4\sqrt{2x-4}+4}$$
$$+\frac{1}{\sqrt{2}}\sqrt{(\sqrt{2x-4})^2-4\sqrt{2x-4}+4}$$
$$\Rightarrow f(x)=\frac{1}{\sqrt{2}}|\sqrt{2x-4}+2|+\frac{1}{\sqrt{2}}|\sqrt{2x-4}-2|$$
$$\Rightarrow f(x)=\begin{cases}\frac{1}{\sqrt{2}}\times 4, & \text{if } \sqrt{2x-4}<2\\ \sqrt{2}\cdot\sqrt{2x-4}, & \text{if } \sqrt{2x-4}\geq 2\end{cases}$$
$$\Rightarrow f(x)=\begin{cases}2\sqrt{2}, & \text{if } x\in[2,4)\\ 2\sqrt{x-2}, & \text{if } x\in[4,\infty)\end{cases}$$
Hence, $f'(x)=\begin{cases}0 & \text{if } x\in[2,4)\\ \frac{1}{\sqrt{x-2}} & \text{if } x\in(4,\infty)\end{cases}$

14 **(c)**

We have,

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x}=\lim_{x\to 0}x\sin\frac{1}{x}=0$$
So, $f(x)$ is differentiable at $x=0$ such that $f'(0)=0$

For $x\neq 0$, we have

$$f'(x)=2x\sin\left(\frac{1}{x}\right)+x^2\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$
$$\Rightarrow f'(x)=2x\sin\frac{1}{x}-\cos\frac{1}{x}$$
$$\Rightarrow \lim_{x\to 0}f'(x)=\lim_{x\to 0}2x\sin\frac{1}{x}-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)=0-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$$
Since $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exist

$\therefore \lim_{x\to 0}f'(x)$ does not exist

Hence, $f'(x)$ is not continuous at $x=0$

15 **(c)**

We have,

$$f(x)=\begin{cases}\frac{x}{\sqrt{x^2}}, & x\neq 0\\ 0, & x=0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}=\begin{cases}1, & x>0\\ -1, & x<0\\ 0, & x=0\end{cases}$$

CLearly, $f(x)$ is not continuous at $x=0$

17 **(c)**

Given, $\lim_{x\to 0}\left[(1+3x)^{\frac{1}{x}}\right]=k$

$\therefore\quad e^3=k$

18 **(b)**

For $x>2$, we have

$$f(x)=\int_0^x\{5+|1-t|\}dt$$

$$\Rightarrow f(x)=\int_0^1(5+(1-t)dt+\int_1^x(5-(1-t)\,dt$$

$$\Rightarrow f(x)=\int_0^1(6-t)dt+\int_1^x(4+t)dt$$

$$\Rightarrow f(x)=\left[6t-\frac{t^2}{2}\right]_0^1+\left[4t+\frac{t^2}{2}\right]_1^x$$

$$\Rightarrow f(x)=1+4x+\frac{x^2}{2}$$

Thus, we have

$$f(x)=\begin{cases}5x+1, & \text{if } x\leq 2\\ \frac{x^2}{2}+4x+1, & \text{if } x>2\end{cases}$$

Clearly, $f(x)$ is everywhere continuous and differentiable except possibly at $x=2$

Now,

$$\lim_{x\to 2^-}f(x)\ =\ \lim_{x\to 2}5x+1\ =11$$

and,

$$\lim_{x\to 2^+}f(x)\ =\ \lim_{x\to 2}\left(\frac{x^2}{2}+4x+1\right)\ =11$$

$$\therefore\lim_{x\to 2^-}f(x)\ =\ \lim_{x\to 2^+}f(x)$$

So, $f(x)$ is continuous at $x=2$

Also, we have (LHD at $x=2$) $=\lim_{x\to 2^-}f'(x)=\lim_{x\to 2}5=5$

19 **(b)**

The given function is clearly continuous at all points except possibly at $x=\pm 1$

For $f(x)$ to be continuous at $x=1$, we must have

$$\lim_{x\to 1^-}f(x)\ =\ \lim_{x\to 1^+}f(x)\ =f(1)$$

$$\Rightarrow\lim_{x\to 1}ax^2+b\ =\ \lim_{x\to 1}\frac{1}{|x|}$$

$\Rightarrow a + b = 1$ ...(i)

Clearly, $f(x)$ is differentiable for all x, except possibly at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x = 1$ only

For $f(x)$ to be differentiable at $x = 1$, we must have

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{x\to 1} \frac{ax^2 + b - 1}{x - 1} = \lim_{x\to 1} \frac{\frac{1}{|x|} - 1}{x - 1}$$

$$\Rightarrow \lim_{x\to 1} \frac{ax^2 - a}{x - 1} = \lim_{x\to 1} \frac{\frac{1}{x} - 1}{x - 1} \quad [\because a + b = 1 \; \therefore b - 1 = -a]$$

$$\Rightarrow \lim_{x\to 1} a(x + 1) = \lim_{x\to 1} \frac{-1}{x}$$

$\Rightarrow 2a = -1 \Rightarrow a = -1/2$

Putting $a = -1/2$ in (i), we get $b = 3/2$

20 **(c)**

At no point, function is continuous

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	C	C	A	A	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	C	C	C	C	B	B	C