DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 5

Topic :- CONTINUITY AND DIFFERENTIABILITY

2 **(c)**

$f'(2^+) = \lim_{x\to 2^+}\left(\frac{f(x)-f(2)}{x-2}\right)$

$= \lim_{x\to 2^+}\frac{3x+4-(6+4)}{x-2} = \lim_{x\to 2}\frac{3x-6}{x-2} = 3$

3 **(a)**

Here, $f(x) = \begin{cases} \sin x, x > 0 \\ 0,\ x = 0 \\ -\sin x,\ \ x < 0 \end{cases}$

$\text{RHD} = \lim_{h\to 0}\frac{\sin|0+h| - \sin(0)}{h}$

$= \lim_{h\to 0}\frac{\sin h}{h} = 1$

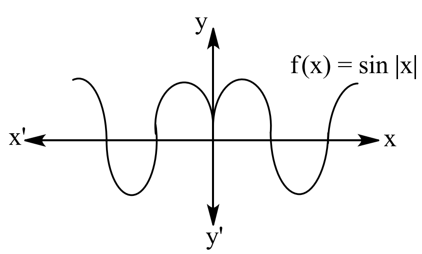
$\text{LHD} = \lim_{h\to 0}\frac{\sin|(0-h)| - \sin(0)}{-h}$

$= \frac{-\sin h}{h} = -1$

$\therefore$ LHD $\neq$ RHD at $x = 0$

$\therefore$ $f(x)$ is not derivable at $x = 0$

Alternate



It is clear from the graph that $f(x)$ is not differentiable at $x = 0$

4 **(b)**

We have,

$f(x) = \sum_{n=0}^{\infty}\frac{x^n}{n!}(\log_e a)^n$

$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{(x \log_e a)^n}{n!} = \sum_{n=0}^{\infty} \frac{\left(\log_e a^x\right)^n}{n!}$

$\Rightarrow f(x) = e^{\log_e a^x} = a^x$, which is everywhere continuous and differentiable

5 **(c)**

$f(x) = [x] \cos \left[\frac{2x-1}{2}\right] \pi$

Since, $[x]$ is always discontinuous at all integer value, hence $f(x)$ is discontinuous for all integer value

6 **(c)**

The function f is clearly continuous for $|x| > 1$

We observe that

$\lim_{x \to -1^+} f(x) = 1, \quad \lim_{x \to -1^-} f(x) = \frac{1}{4}$

Also, $\lim_{x \to \frac{1}{n}^+} f(x) = \frac{1}{n^2}$ and, $\lim_{x \to \frac{1}{n}^-} f(x) = \frac{1}{(n+1)^2}$

Thus, f is discontinuous for $x = \pm \frac{1}{n}, n = 1, 2, 3, \ldots$

7 **(c)**

Since, $|f(x) - f(y)| \le (x-y)^2$

$\Rightarrow \lim_{x \to y} \frac{|f(x) - f(y)|}{|x-y|} \le \lim_{x \to y} |x-y|$

$\Rightarrow |f'(y)| \le 0$

$\Rightarrow f'(y) = 0$

$\Rightarrow f(y) = \text{constant}$

$\Rightarrow f(y) = 0 \Rightarrow f(1) = 0 \quad [\because f(0) = 0, \text{ given}]$

8 **(b)**

Since $\phi(x) = 2x^3 - 5$ is an increasing function on $(1, 2)$ such that $\phi(1) = -3$ and $\phi(2) = 11$

Clearly, between -3 and 11 there are thirteen points where $f(x) = [2x^3 - 5]$ is discontinuous

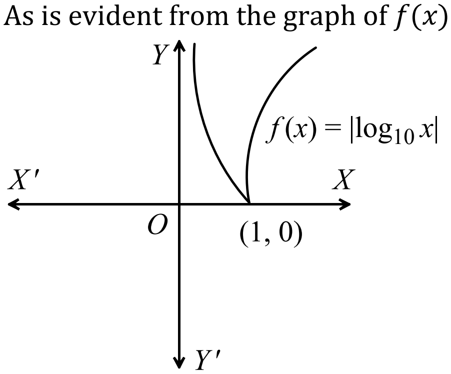
9 **(c)**

Clearly, $[x^2 + 1]$ is discontinuous at $x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$

Note that it is right continuous at $x = 1$ but not left continuous at $x = 3$

10 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$$f''(1^+) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log_{10}(1+h) - 0}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$$

$$f''(1^-) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{h}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{\log_{10}(1-h)}{h} = \lim_{h\to 0} \frac{\log_e(1-h)}{h \log_e 10} = -\log_{10} e$$

11 **(b)**

It can be easily seen from the graph of $f(x) = |\cos x|$ that it is everywhere continuous but not differentiable at odd multiples of $\pi/2$

12 **(d)**

We have,

$$\lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0} \frac{4-h-4}{|4-h-4|} + a$$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} -\frac{h}{h} + a = a - 1$$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} \frac{4+h-4}{|4+h-4|} + b = b + 1$$

and, $f(4) = a + b$

Since $f(x)$ is continuous at $x = 4$. Therefore,

$$\lim_{x\to 4^-} f(x) = f(4) = \lim_{x\to 4^+} f(x)$$

$\Rightarrow a - 1 = a + b = b + 1 \Rightarrow b = -1$ and $a = 1$

13 **(b)**

We have,

$$f(x) = \begin{cases} \dfrac{2^x - 1}{\sqrt{1+x} - 1}, & -1 \le x < \infty, \quad x \ne 0 \\ k, & x = 0 \end{cases}$$

Since, $f(x)$ is continuous everywhere

$\therefore \quad \lim_{x\to 0^-} f(x) = f(0) \quad$ ...(i)

Now, $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} \frac{2^{(0-h)} - 1}{\sqrt{1+(0-h)} - 1}$

$$= \lim_{h \to 0} \frac{2^{-h} - 1}{\sqrt{1-h} - 1}$$

$$= \lim_{h \to 0} \frac{-2^{-h} \log_e 2}{\frac{-1}{2\sqrt{1-h}}} \text{ [by L' Hospital's rule]}$$

$$= 2 \lim_{h \to 0} 2^{-h} \log_e 2\sqrt{1-h}$$

$$= 2 \log_e 2$$

From Eq. (i),

$f(0) = 2 \log_e 2 = \log_e 4$

15 **(b)**

We have,

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(-h) = \lim_{h \to 0} \frac{e^{-1/h} - 1}{e^{-1/h} + 1} = -1$$

and,

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(h) = \lim_{x \to 0} \frac{e^{1/h} - 1}{e^{1/h} + 1} = \lim_{h \to 0} \frac{e^{-1/h}}{e^{-1/h}} = 1$$

$$\therefore \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

Hence, $f(x)$ is not continuous at $x = 0$

16 **(c)**

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} 1 + (2 - h) = 3$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} 5 - (2 + h) = 3, \quad f(2) = 3$$

Hence, f is continuous at $x = 2$

Now, $Rf''(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$$= \lim_{h \to 0} \frac{5 - (2 + h) - 3}{h} = -1$$

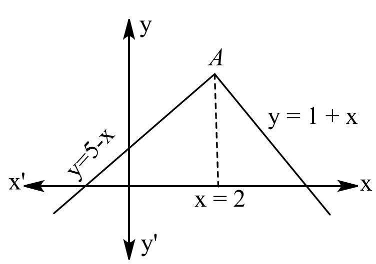
$$Lf''(2) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$$

$$= \lim_{h \to 0} \frac{1 + (2 - h) - 3}{-h} = 1$$

$\therefore \; Rf''(2) \neq Lf''(2)$

$\therefore \; f$ is not differentiable at $x = 2$

Alternate



It is clear from the graph that $f(x)$ is continuous everywhere also it is differentiable everywhere except at $x = 2$

17 **(d)**

We have,

$f(x+y) = f(x)f(y)$ for all $x, y \in R$

Putting $x = 1, y = 0$, we get

$f(0) = f(1)f(0) \Rightarrow f(0)(1 - f(1)) = 0$

$\Rightarrow f(1) = 1 \quad [\because f(0) \neq 0]$

Now,

$f'(1) = 2$

$$\Rightarrow \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = 2$$

$$\Rightarrow \lim_{h\to 0} \frac{f(1)f(h) - f(1)}{h} = 2$$

$$\Rightarrow f(1) \lim_{h\to 0} \frac{f(h) - 1}{h} = 2$$

$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 2 \quad$ [Using $f(1) = 1$] ...(i)

$$\therefore f'(4) = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$$

$$\Rightarrow f'(4) = \lim_{h\to 0} \frac{f(4)f(h) - f(4)}{h}$$

$$\Rightarrow f'(4) = \left\{\lim_{h\to 0} \frac{f(h) - 1}{h}\right\} f(4)$$

$\Rightarrow f'(4) = 2\, f(4) \quad$ [From (i)]

$\Rightarrow f'(4) = 2 \times 4 = 8$

18 **(d)**

We have,

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = 1$ and $g(1) = 0$

So, $g(x)$ is not continuous at $x = 1$ but $\lim_{x\to 1} g(x)$ exists

We have,

$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} [1-h] = 0$

and,

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} [1+h] = 1$

So, $\lim_{x\to 1} f(x)$ does not exist and so $f(x)$ is not continuous at $x = 1$

We have, $gof(x) = g(f(x)) = g([x]) = 0$, for all $x \in R$

So, gof is continuous for all x

We have,

$fog(x) = f(g(x))$

$$\Rightarrow fog(x) = \begin{cases} f(0), & x \in Z \\ f(x^2), & x \in R - Z \end{cases}$$

$$\Rightarrow fog(x) = \begin{cases} 0, & x \in Z \\ [x^2], & x \in R - Z \end{cases}$$

Which is clearly not continuous

19 **(d)**

At $x = 1$,

RHD $= \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$

$$= \lim_{h \to 0} \frac{2 - (1+h) - (2-1)}{h} = -1$$

LHD $= \lim_{h \to 0^-} \frac{f(1-h) - f(1)}{-h}$

$$= \lim_{h \to 0} \frac{(1-h) - (2-1)}{-h} = 1$$

$\therefore$ LHD $\neq$ RHD

20 **(d)**

Given, $f(x) = |x| + \frac{|x|}{x}$

Let $f_1(x) = |x|$, $f_2(x) = \frac{|x|}{x}$

1. LHL $= \lim_{x \to 0^-} f_1(x) = \lim_{x \to 0^-} |x| = 0$

And RHL $\lim_{x \to 0^+} f_1(x) = \lim_{x \to 0^+} |x| = 0$

Here, LHL=RHL=$f(0)$, $f_1(x)$ is continuous

2. LHL $= \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{h \to 0} \frac{|0-h|}{0-h} = -1$

RHL $= \lim_{x \to 0^+} \frac{|x|}{x} = \lim_{h \to 0} \frac{|0+h|}{h} = 1$

$\therefore$ LHL $\neq$ RHL, $f_2(x)$ is discontinuous

Hence, $f(x)$ is discontinuous at $x = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	C	C	C	B	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	B	C	B	C	D	D	D	D