DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth **SOLUTIONS** **SUBJECT : MATHS**
DATE : **DPP NO. : 4**

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(a)**

We have, $f(x)=\begin{cases} x^2, & x\geq 0 \\ -x^2, & x<0 \end{cases}$

Clearly, $f(x)$ is differentiable for all $x>0$ and for all $x<0$. So, we check the differentiable at $x=0$

Now, (RHD at $x=0$)

$\left(\frac{d}{dx}(x)^2\right)_{x=0}=(2x)_{x=0}=0$

And (LHD at $=0$)

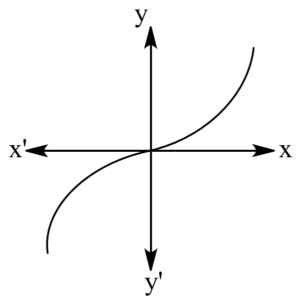
$\left(\frac{d}{dx}(-x)^2\right)_{x=0}=(-2x)_{x=0}=0$

$\therefore$ (LHD at $x=0$)=(RHD at $x=0$)

So, $f(x)$ is differentiable for all x *ie*, the set of all points where $f(x)$ is differentiable is $(-\infty,\infty)$

Alternate

It is clear from the graph $f(x)$ is differentiable everywhere.



2 **(a)**

Since, $f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=10$

$\Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=10$

$\Rightarrow f(0)\left(\lim_{h\to 0}\frac{f(h)-1}{h}\right)=10$ ...(i)

[$\because$ $f(0+h)=f(0)f(h)$, given]

Now, $f(0)=f(0)f(0)$

$\Rightarrow f(0)=1$

$\therefore$ From Eq. (i)

$\lim_{h\to 0} \frac{f(h)-1}{h} = 10$ ...(ii)

Now, $f'(6) = \lim_{h\to 0} \frac{f(6+h)-f(6)}{h}$

$= \lim_{x\to 0} \left(\frac{f(h)-1}{h}\right) f(6)$ [from Eq. (ii)]

$= 10 \times 3 = 30$

3 **(a)**

We have,

$f'(a^+) = \lim_{x\to a^+} \frac{f(x)-f(0)}{x-a}$

$\Rightarrow f'(a^+) = \lim_{x\to a^+} \frac{|x-a|\phi(x)}{x-a}$

$\Rightarrow f'(a^+) = \lim_{x\to a} \frac{(x-a)}{(x-a)} \phi(x)$ $[\because x > a \therefore |x-a| = x-a]$

$\Rightarrow f'(a^+) = \lim_{x\to a} \phi(x)$

$\Rightarrow f'(a^+) = \phi(a)$ [$\because \phi(x)$ is continuous at $x = a$]

and,

$f'(a^-) = \lim_{x\to a^-} \frac{f(x)-f(0)}{x-a}$

$\Rightarrow f'(a^-) = \lim_{x\to a^-} \frac{|x-a|\phi(x)}{x-a}$

$\Rightarrow f'(a^-) = \lim_{x\to a} \frac{(x-a)\phi(x)}{(x-a)}$ $[\because x < a \therefore |x-a| = -(x-a)]$

$\Rightarrow f'(a^-) = -\lim_{x\to a} \phi(x)$

$\Rightarrow f'(a^-) = -\phi(a)$ [$\because \phi(x)$ is continuous at $x = a$]

4 **(b)**

$\text{LHL} = \lim_{h\to 0} (0-h)_e^{-\left(\frac{1}{|-h|}+\frac{1}{(-h)}\right)} = \lim_{h\to 0}(-h) = 0$

$\text{RHL} = \lim_{h\to 0} (0+h)_e^{-\left(\frac{1}{|h|}+\frac{1}{(h)}\right)} = \lim_{h\to 0} \frac{h}{e^{2/h}} = 0$

LHL=RHL $= f(0)$

Therefore, $f(x)$ is continuous for all x

Differentiability at $x = 0$

$Lf'(0) = \lim_{h\to 0} \frac{(-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}}{(-h)-0} = 1$

$Rf'(0) = \lim_{h\to 0} \frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)-0}}{h-0}$

$= \lim_{h\to 0} \frac{1}{e^{2/h}} = 0$

$\Rightarrow Rf'(0) Lf'(0)$

Therefore, $f(x)$ is not differentiable at $x = 0$

5 **(d)**

We have,

$f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$

Clearly, f is continuous but not differentiable at $x = 0$

Now,

$f(|x|) = 2|x| + 1$ for all x

Clearly, $f(|x|)$ is everywhere continuous but not differentiable at $x = 0$

7 **(c)**

We have,

$f(x) = |x - 0.5| + |x - 1| + \tan x, 0 < x < 2$

$\Rightarrow f(x) = \begin{cases} -2x + 1.5 + \tan x, & 0 < x < 0.5 \\ 0.5 + \tan x, & 0.5 \leq x < 1 \\ 2x - 1.5 + \tan x, & 1 \leq x < 2 \end{cases}$

It is evident from the above definition that

$Lf'(0.5) \neq Rf'(0.5)$ and $Lf'(1) \neq Rf'(1)$

Also, the function is not continuous at $= \pi/2$. So, it cannot be differentiable thereat

8 **(d)**

Given, $f(x) = \begin{cases} \log_{(1-3x)}(1 + 3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \frac{\log(1 + 3x)}{\log(1 - 3x)}$

$= -\lim\limits_{x \to 0} \frac{\log(1 + 3x)}{3x} \cdot \frac{(-3x)}{\log(1 - 3x)}$

$= -1$

And $f(0) = k$

$\because f(x)$ is continuous at $x = 0$

$\therefore \quad k = -1$

9 **(d)**

Since $f(x)$ is differentiable at $x = c$. Therefore, it is continuous at $x = c$

Hence, $\lim\limits_{x \to c} f(x) = f(c)$

10 **(a)**

Given, $f(x) = ae^{|x|} + b\,|x|^2$

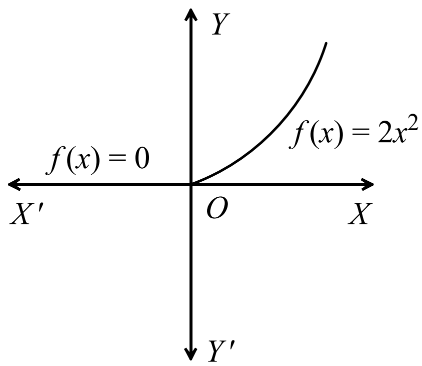
We know $e^{|x|}$ is not differentiable at $x = 0$ and $|x|^2$ is differentiable at $x = 0$

$\therefore \ f(x)$ is differentiable at $x = 0$, if $a = 0$ and $b \in R$

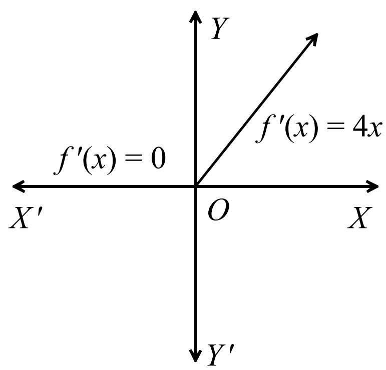
11 **(a)**

We have,

$f(x) = \begin{cases} (x - x)(-x) = 0, x < 0 \\ (x + x)x = 2x^2, x \geq 0 \end{cases}$



(i)



(ii)

As is evident from the graph of $f(x)$ that it is continuous and differentiable for all x

Also, we have

$$f''(x)=\begin{cases}0, x<0\\4x, x\geq 0\end{cases}$$

Clearly, $f''(x)$ is continuous for all x but it is not differentiable at $x=0$

12 **(b)**

Given, $f(x)=\begin{cases}\frac{x-1}{2x^2-7x+5}, & x\neq 1\\ -\frac{1}{3}, & x=1\end{cases}$

$$f(x)=\begin{cases}\frac{1}{2x-5}, & x\neq 1\\ -\frac{1}{3}, & x=1\end{cases}$$

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2(1+h)-5}-\left(-\frac{1}{3}\right)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2h-3}+\frac{1}{3}}{h}=\lim_{h\to 0}\frac{3+2h-3}{3h(2h-3)}=-\frac{2}{9}$$

$$Lf'(1)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2(1-h)-5}-\left(-\frac{1}{3}\right)}{-h}$$

$$=\lim_{h\to 0}-\frac{2}{3(2h+3)}=-\frac{2}{9}$$

$$\therefore\ f'(1)=-\frac{2}{9}$$

13 **(b)**

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{f(1+h)}{h}-\lim_{h\to 0}\frac{f(1)}{h}$$

Given, $\lim\limits_{h\to 0}\frac{f(1+h)}{h}=5$

So, $\lim\limits_{h\to 0}\frac{f(1)}{h}$ must be finite as $f'(1)$ exist and $\lim\limits_{h\to 0}\frac{f(1)}{h}$ can be finite only, if $f(1)=0$ and $\lim\limits_{h\to 0}\frac{f(1)}{h}=0$

So, $f'(1)=\lim\limits_{h\to 0}\frac{f(1+h)}{h}=5$

14 **(c)**

Since, $f(x)$ is continuous for every value of R except {-1, -2}. Now, we have to check that points

At $x=-2$

$$\text{LHL}=\lim_{h\to 0}\frac{(-2-h)+2}{(-2-h)^2+3(-2-h)+2}$$

$$=\lim_{h\to 0}\frac{-h}{h^2+h}=-1$$

$$\text{RHL}=\lim_{h\to 0}\frac{(-2+h)+2}{(-2+h)^2+3(-2+h)+2}$$

$$=\lim_{h\to 0}\frac{h}{h^2-h}=-1$$

$\Rightarrow$ LHL=RHL=$f(-2)$

$\therefore$ It is continuous at $x=-2$

Now, check for $x=-1$

$$\text{LHL}=\lim_{h\to 0}\frac{(-1-h)+2}{(-1-h)^2+3(-1-h)+2}$$

$$=\lim_{h\to 0}\frac{1-h}{h^2-h}=\infty$$

$$\text{RHL}=\lim_{h\to 0}\frac{(-1+h)+2}{(-1+h)^2+3(-1+h)+2}$$

$$=\lim_{h\to 0}\frac{1+h}{h^2+h}=\infty$$

$\Rightarrow$ LHL=RHL $\neq f(-1)$

$\therefore$ It is not continuous at $x=-1$

The required function is continuous in $R-\{-1\}$

15 **(d)**

$$f(0) = \lim_{x\to 0} \frac{(e^x - 1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1 + \frac{x}{4}\right)}$$

$$\Rightarrow \quad \lim_{x\to 0}\left(\frac{e^x - 1}{x}\right)^2 \cdot \frac{\frac{x}{a}.a}{\sin\frac{x}{a}} \cdot \frac{\frac{x}{4}.4}{\log\left(1 + \frac{x}{4}\right)} = 12$$

$\Rightarrow \quad 1^2.a.4 = 12$

$\Rightarrow \quad a = 3$

16 **(b)**

We have,

$$f(x) = \frac{x}{1 + x} + \frac{x}{(x + 1)(2x + 1)} + \frac{x}{(2x + 1)(3x + 1)} + \dots\infty$$

$$\Rightarrow f(x) = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{x}{((r - 1)x + 1)\,(rx + 1)}, \text{ for } x \neq 0$$

$$\Rightarrow f(x) = \lim_{n\to\infty}\sum_{r=1}^{n}\left\{\frac{1}{(r - 1)x + 1} - \frac{1}{rx + 1}\right\}, \text{ for } x \neq 0$$

$$\Rightarrow f(x) = \lim_{n\to\infty}\left\{1 - \frac{2}{nx + 1}\right\} = 1 \text{ , for } x \neq 0$$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$

17 **(b)**

If possible, let $f(x) + g(x)$ be continuous. Then, $\{f(x) + g(x)\} - f(x)$ must be continuous

$\Rightarrow g(x)$ must be continuous

This is a contradiction to the given fact that $g(x)$ is discontinuous

Hence, $f(x) + g(x)$ must be discontinuous

18 **(c)**

We have,

$f(x + y) = f(x)f(y)$ for all $x, y \in R$

$\therefore f(0) = f(0)f(0)$

$\Rightarrow f(0)\{f(0) - 1\} = 0$

$\Rightarrow f(0) = 1 \qquad [\because f(0) \neq 1]$

Now,

$f'(0) = 0$

$$\Rightarrow \lim_{h\to 0}\frac{f(0 + h) - f(0)}{h} = 2$$

$$\Rightarrow \lim_{h\to 0}\frac{f(h) - 1}{h} = 2 \qquad [\because f(0) = 1] \quad \dots(i)$$

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h}$ [$\because f(x+y) = f(x)f(y)$]

$\Rightarrow f'(x) = f(x)\left\{\lim_{h\to 0} \frac{f(h)-1}{h}\right\} = 2f(x)$ [Using (i)]

19 **(b)**

We have,

$$f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{x^2}{2} = x, & x > 0 \\ 0, & x = 0 \\ \frac{x^2}{-x} = -x, & x < 0 \end{cases}$$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0} -x = 0,\ \lim_{x\to 0^+} f(x) = \lim_{x\to 0} x = 0$ and $f(0) = 0$

So, $f(x)$ is continuous at $x = 0$. Also, $f(x)$ is continuous for all other values of x

Hence, $f(x)$ is everywhere continuous

Clearly, $Lf'(0) = -1$ and $Rf'(0) = 1$

Therefore, $f(x)$ is not differentiable at $x = 0$

20 **(b)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) \Rightarrow f(0) = 2$ ...(i)

Now, using L' Hospital's rule, we have

$\lim_{x\to 0} \frac{\int_0^x f(u)\ du}{x} = \lim_{x\to 0} \frac{f(x)}{1} = f(0)$ [$\because f(x)$ is continuous at $x = 0$]

$\Rightarrow \lim_{x\to 0} \frac{\int_0^x f(u)\ du}{x} = 2$ [Using (i)]

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	A	B	D	A	C	D	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	B	C	D	B	B	C	B	B