DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 2

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

Clearly, $f(x)$ is differentiable for all non-zero values of x. For $x \neq 0$, we have

$$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

Now,

$$(\text{LHD at } x=0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{x-0}$$

$$\Rightarrow (\text{LHD at } x=0) = \lim_{h\to 0} \frac{\sqrt{1-e^{-h^2}}}{-h} = \lim_{h\to 0} -\frac{\sqrt{1-e^{-h^2}}}{h}$$

$$\Rightarrow (\text{LHD at } x=0) = -\lim_{h\to 0} \sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = -1$$

$$\text{and, } (\text{RHD at } x=0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{\sqrt{1-e^{-h^2}}-0}{h}$$

$$\Rightarrow (\text{RHD at } x=0) = \lim_{h\to 0} \sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = 1$$

So, $f(x)$ is not differentiable at $x=0$

Hence, the set of points of differentiability of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

2 **(c)**

Since $f(x)$ is continuous at $x=0$

$$\therefore f(0) = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$$

3 **(d)**

For $f(x)$ to be continuous everywhere, we must have,

$$f(0) = \lim_{x\to 0} f(x)$$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2} \quad \left[\text{Form } \frac{0}{0}\right]$$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{\frac{7}{8}(256-7x)^{-\frac{7}{8}}}{(5x+32)^{-4/5}} = \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$$

4 **(b)**

We have,

$$f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$$

$$\therefore \text{(LHD at } x=0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} -\frac{x^3}{x} = 0$$

and,

$$\therefore \text{(RHD at } x=0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x^3}{x} = 0$$

Clearly, (LHD at $x = 0$) = (RHD at $x = 0$)

Hence, $f(x)$ is differentiable at $x = 0$ and its derivative at $x = 0$ is 0

5 **(a)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{4^x-1}{x}\right)^3 \times \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{ax^2}{\log\left(1+\frac{1}{3}x^2\right)}$$

$$= (\log 4)^3 .1. a \lim_{x\to 0}\left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \dots}\right)$$

$$= 3a\,(\log 4)^3$$

$$\because \quad \lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow\ 3a\,(\log 4)^3 = 9(\log 4)^3$$

$$\Rightarrow \quad a = 3$$

6 **(d)**

We have,

$f(x) = |[x]x|$ for $-1 < x \leq 2$

$$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \leq x < 1 \\ x, & 1 \leq x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non-differentiable at $x = 2$

7 **(c)**

Given, $f(x) = [x^3 - 3]$

Let $g(x) = x^3 - x$ it is in increasing function

$\therefore\ g(1) = 1 - 3 = -2$

and $g(2) = 8 - 3 = 5$

Here, $f(x)$ is discontinuous at six points

8 **(b)**

Given, $y = \cos^{-1}\cos(x-1),\ \ x > 0$

$\Rightarrow\ \ y = x - 1, \qquad 0 \leq x - 1 \leq \pi$

$\therefore\ y = x - 1, \qquad 1 \le x \le \pi + 1$

At $x = \frac{5\pi}{4} \in [1, \pi + 1]$

$\Rightarrow \dfrac{dy}{dx} = 1 \ \Rightarrow\ \left(\dfrac{dy}{dx}\right)_{x=\frac{5\pi}{4}} = 1$

9 **(d)**

We have,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x) + f(h) - f(x)}{h} \qquad [\because f(x+y) = f(x) + f(y)]$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(h)}{h} = \lim_{h\to 0} \frac{h^2 g(h)}{h}$$

$$\Rightarrow f'(x) = 0 \times g(0) = 0 \qquad \left[\begin{array}{l} \because g \text{ is continuous} \\ \therefore \lim\limits_{h\to 0} g(h) = g(0) \end{array}\right]$$

10 **(b)**

Using Heine's definition of continuity, it can be shown that $f(x)$ is everywhere discontinuous

11 **(b)**

For $x \ne -1$, we have

$f(x) = 1 - 2x + 3x^2 - 4x^3 + \dots\infty$

$\Rightarrow f(x) = (1 + x)^{-2} = \dfrac{1}{(1 + x)^2}$

Thus, we have

$$f(x) = \begin{cases} \dfrac{1}{(1+x)^2}, & x \ne -1 \\ 1, & x = -1 \end{cases}$$

We have, $\lim\limits_{x\to -1^-} f(x) \to \infty$ and $\lim\limits_{x\to -1^-} f(x) \to \infty$

So, $f(x)$ is not continuous at $x = -1$

Consequently, it is not differentiable there at

12 **(b)**

At $x = a$,

LHL $= \lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a} 2a - x = a$

And RHL $= \lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a} 3x - 2a = a$

And $f(a) = 3(a) - 2a = a$

$\therefore$ LHL=RHL $= f(a)$

Hence, it is continuous at $x = a$

Again, at $x = a$

LHD $= \lim\limits_{h\to 0} \frac{f(a-h) - f(a)}{-h}$

$$= \lim_{h \to 0} \frac{2a - (a - h) - a}{-h} = -1$$

and RHD $= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$= \lim_{h \to 0} \frac{3(a+h) - 2a - a}{h} = 3$$

$\therefore$ LHD $\neq$ RHD

Hence, it is not differentiable at $x = a$

13 **(b)**

We have,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x)f(h) - f(x)}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{f(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{1 + (\sin 2h)g(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{\sin 2h}{h} \times \lim_{h \to 0} g(h) = 2f(x)g(0)$$

14 **(c)**

If $-1 \le x \le 1$, then $0 \le x \sin \pi x \le 1/2$

$\therefore f(x) = [x \sin \pi x] = 0$, for $-1 \le x \le 1$

If $1 < x < 1 + h$, where h is a small positive real number, then

$\pi < \pi x < \pi + \pi h \Rightarrow -1 < \sin \pi x < 0 \Rightarrow -1 < x \sin \pi x < 0$

$\therefore f(x) = [x \sin \pi x] = -1$ in the right neighbourhood of $x = 1$

Thus, $f(x)$ is constant and equal to zero in $[-1, 1]$ and so $f(x)$ is differentiable and hence continuous on $(-1, 1)$

At $x = 1$, $f(x)$ is discontinuous because

$\Rightarrow \lim_{x \to 1^-} f(x) = 0$ and $\lim_{x \to 1^+} f(x) = -1$

Hence, $f(x)$ is not differentiable at $x = 1$

15 **(d)**

We have,

$$(\text{LHD at } x = 0) = \left\{\frac{d}{dx}(1)\right\}_{x=0} = 0$$

$$(\text{RHD at } x = 0) = \left\{\frac{d}{dx}(1 + \sin x)\right\}_{x=0} = \cos 0 = 1$$

Hence, $f'(x)$ at $x = 0$ does not exist

16 **(c)**

Here, $f'(x)=\begin{cases}2bx+a, & x\geq -1\\ 2a, & x<-1\end{cases}$

Given, $f'(x)$ is continuous everywhere

$\therefore \lim\limits_{x\to -1^+} f(x)=\lim\limits_{x\to -1^-} f(x)$

$\Rightarrow -2b+a=-2a$

$\Rightarrow 3a=2b$

$\Rightarrow a=2, \quad b=3$

or $a=-2, \; b=-3$

17 **(b)**

We have,

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\log\cos x}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\log(1-1+\cos x)}{\log(1+x^2)}\cdot\frac{1-\cos x}{1-\cos x}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\log\{1-(1-\cos x)\}}{1-\cos x}\cdot\frac{1-\cos x}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=-\lim_{x\to 0}\log\frac{[1-(1-\cos x)]}{-(1-\cos x)}\times\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\times\frac{x^2}{\log(1+x^2)}$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=-\frac{1}{2}$$

Hence, $f(x)$ is differentiable and hence continuous at $x=0$

18 **(a)**

Since $f(x)$ is continuous at $x=1$. Therefore,

$\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x) \Rightarrow A-B=3 \Rightarrow A=3+B$ ...(i)

If $f(x)$ is continuous at $x=2$, then

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x) \Rightarrow 6=4B-A$ ...(ii)

Solving (i) and (ii) we get $B=3$

As $f(x)$ is not continuous at $x=2$. Therefore, $B\neq 3$

Hence, $A=3+B$ and $B\neq 3$

19 **(a)**

We have,

$$f(x)=\begin{cases}x-4, & x\geq 4\\ -(x-4), & 1\leq x<4\\ (x^3/2)-x^2+3x+(1/2), & x<1\end{cases}$$

Clearly, $f(x)$ is continuous for all x but it is not differentiable at $x=1$ and $x=4$

20 **(a)**

It is given that $f(x)$ is continuous at $x=1$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x \to 1^-} a[x+1] + b[x-1] = \lim_{x \to 1^+} a[x+1] + b[x-1]$

$\Rightarrow a - b = 2a + 0 \times b$

$\Rightarrow a + b = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	D	B	A	D	C	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	C	D	C	B	A	A	A