DPP
DAILY PRACTICE PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 9

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

If a function $f(x)$ is continuous at $x=a$, then it may or may not be differentiable at $x=a$

$\therefore$ Option (b) is correct

2 **(c)**

Let $f(x)=|x-1|+|x-3|$

$$=\begin{cases} x-1 \quad +x-3 \quad ,x\geq 3 \\ x-1+3-x, \quad 1\leq x<3 \\ 1-x \quad +3-x, \quad x\leq 1 \end{cases}$$

$$=\begin{cases} 2x-4, x\geq 3 \\ 2, 1\leq x<3 \\ 4-2x, x\leq 1 \end{cases}$$

At $x=2$, function is

$f(x)=2$

$\Rightarrow f'(x)=0$

3 **(d)**

We have,

$$f(x)=\begin{cases} (x+1)\, e^{-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1), \quad x<0 \\ (x+1)\, e^{-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)e^{-2/x}, x>0 \end{cases}$$

Clearly, $f(x)$ is continuous for all $x\neq 0$

So, we will check its continuity at $x=0$

We have,

$$(\text{LHL at } x=0)=\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(x+1) = 1$$

$$(\text{RHL } at\ x=0)=\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x+1)\ e^{-2/x}=\lim_{x\to 0}\frac{x+1}{e^{2/x}}=0$$

$$\therefore \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+ f(x)}$$

So, $f(x)$ is not continuous at $x=0$

Also, $f(x)$ assumes all values from $f(-2)$ to $f(2)$ and $f(2)=3/e$ is the maximum value of $f(x)$

4 **(c)**

Since, it is a polynomial function, so it is continuous for every value of x except at $x=2$

$\text{LHL} = \lim_{x\to 2^-} x - 1$

$= \lim_{h\to 0} 2 - h - 1 = 1$

$\text{RHL} = \lim_{x\to 2^{\mp}} 2x - 3 = \lim_{h\to 0} 2(2+h) - 3 = 1$

And $\quad f(2) = 2(2) - 3 = 1$

$\therefore$ LHL+RHL $= f(2)$

Hence, $f(x)$ is continuous for all real values of x

5 **(c)**

Continuity at $x = 0$

LHL$= \lim_{x\to 0^-} \frac{\tan x}{x} = \lim_{h\to 0} \frac{-\tan h}{-h} = 1$

$\text{RHL} = \lim_{x\to 0^+} \frac{\tan x}{x} = \lim_{h\to 0} \frac{\tan h}{h} = 1$

$\therefore$ LHL=RHL $= f(0) = 1$, it is continuous

Differentiability at $x = 0$

$\text{LHD} = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0} \frac{\frac{\tan(-h)}{-h} - 1}{-h}$

$= \lim_{h\to 0} \frac{+\frac{h^2}{3} + \frac{2h^4}{15} + \dots}{-h} = 0$

$\text{RHD} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\frac{\tan h}{h} - 1}{h}$

$= \lim_{h\to 0} \frac{\frac{h^2}{3} + \frac{2h^4}{15} + \dots}{-h} = 0$

$\therefore$ LHD=RHD

Hence, it is differentiable.

6 **(b)**

We have,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} (x-1) = 0$

and,

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} (x^3 - 1) = 0$. Also, $f(1) = 1 - 1 = 0$

So, $f(x)$ is continuous at $x = 1$

Clearly, $\left(f'(1)\right) = 3$ and $Rf'(1) = 1$

Therefore, $f(x)$ is not differentiable at $x = 1$

7 **(d)**

We have,

$$f(x)=\begin{cases}\dfrac{x^2-x}{x^2-x}=1, & \text{if } x<0 \text{ or } x>1\\ -\dfrac{(x^2-x)}{x^2-x}=-1, & \text{if } 0<x<1\\ 1, & \text{if } x=0\\ -1, & \text{if } x=1\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}1, \text{ if } x\le 0 \text{ or } x>1\\ -1, \text{ if } 0<x\le 1\end{cases}$$

Now,

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0} 1=1$ and, $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0} -1=-1$

Clearly, $\lim\limits_{x\to 0^-} f(x)\neq \lim\limits_{x\to 0^+} f(x)$

So, $f(x)$ is not continuous at $x=0$. It can be easily seen that it is not continuous at $x=1$

8 **(b)**

We have,

$f(x)=|x-1|+|x-3|$

$$\Rightarrow f(x)=\begin{cases}-(x-1)-(x-3), & x<1\\ (x-1)-(x-3), & 1\le x<3\\ (x-1)+(x-3), & x\ge 3\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-2x+4, & x<1\\ 2, & 1\le x<3\\ 2x-4, & x\ge 3\end{cases}$$

Since, $f(x)=2$ for $1\le x<3$. Therefore $f'(x)=0$ for all $x\in(1,3)$

Hence, $f'(x)=0$ at $x=2$

9 **(d)**

We have,

$Lf'(0)=0$ and $Rf'(0)=0+\cos 0°=1$

$\therefore Lf'(0)\neq Rf'(0)$

Hence, $f'(x)$ does not exist at $x=0$

10 **(c)**

Given, $g(x)=\frac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$, $m\neq 0$, n are integers and $|x-1|=\begin{cases}x-1; & x\ge 1\\ 1-x; & x<1\end{cases}$

The left hand derivative of $|x-1|$ at $x=1$ is $p=-1$

Also, $\lim\limits_{x\to 1^+} g(x)=p=-1$

$$\Rightarrow \lim_{h\to 0}\frac{(1+h-1)^n}{\log\cos^m(1+h-1)}=-1$$

$$\Rightarrow \lim_{h\to 0}\frac{h^n}{m\log\cos h}=-1$$

$$\Rightarrow \lim_{h\to 0}\frac{n.h^{n-1}}{m\dfrac{1}{\cos h}(-\sin h)}=-1$$

[using L 'Hospital's rule]

$\Rightarrow \quad \left(\frac{n}{m}\right)\lim_{h\to 0}\frac{h^{n-2}}{\left(\frac{\tan h}{h}\right)} = 1$

$\Rightarrow\ n = 2$ and $\frac{n}{m} = 1$

$\Rightarrow\ m = n = 2$

11 **(c)**

Given, $f(x) = \frac{2x^2+7}{(x^2-1)(x+3)}$

Since, at $x = 1, -1, -3,\ f(x) = \infty$

Hence, function is discontinuous

13 **(a)**

$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0}\ [1-(1-h)^2] = 0$

$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0}\{1+(1+h)^2\} = 2$

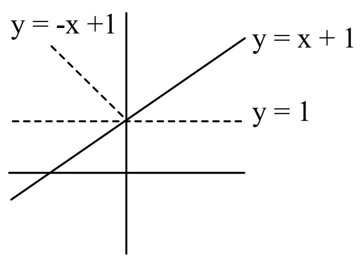
Also, $f(1) = 0$

$\Rightarrow\ \text{RHL} \neq \text{LHL} = f(1)$

Hence, $f(x)$ is not continuous at $x = 1$

14 **(c)**

It is clear from the graph that minimum $f(x)$ is



$f(x) = x + 1, \qquad \forall\ x \in R$

Hence, it is a straight line, so it is differentiable everywhere

15 **(c)**

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\lim_{x\to \frac{\pi^{-1}}{2}} (mx+1) = \lim_{x\to \frac{\pi^+}{2}} (\sin x + n)$

$\Rightarrow \quad m\frac{\pi}{2} + 1 = \sin\frac{\pi}{2} + n$

$\Rightarrow \quad \frac{m\pi}{2} = n$

16 **(a)**

This function is continuous at $x = 0$, then

$\lim_{x\to 0}\frac{\log_e(1+x^2\tan x)}{\sin x^3} = f(0)$

$\Rightarrow\ \lim_{x\to 0}\frac{\log_e\left\{1+x^2\left(x+\frac{x^3}{3}+\ldots\right)\right\}}{x^3-\frac{x^9}{3!}+\frac{x^{15}}{5!}-\ldots} = f(0)$

$$\Rightarrow \lim_{x\to 0} \frac{\log_e(1+x^3)}{x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \dots} = f(0)$$

[neglecting higher power of x in $x^2 \tan x$]

$$\Rightarrow \lim_{x\to 0} \frac{x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \dots}{x^3 + \frac{x^9}{3!} + \frac{x^{15}}{5!} - \dots} = f(0)$$

$\Rightarrow \quad 1 = f(0)$

17 **(a)**

Given, $f(x)$ is continuous at $x = 0$

$\therefore$ Limit must exist

ie, $\lim_{x\to 0} x^p \sin\frac{1}{x} = (0)^p \sin\infty = 0$, when, $0 < p < \infty$ ...(i)

Now, RHD $= \lim_{h\to 0} \frac{h^p \sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} h^{p-1} \sin\frac{1}{h}$

LHD $= \lim_{h\to 0} \frac{(-h)^p \sin\left(-\frac{1}{h}\right) - 0}{-h}$

$= \lim_{h\to 0} (-1)^p h^{p-1} \sin\frac{1}{h}$

Since, $f(x)$ is not differentiable at $x = 0$

$\therefore \quad p \le 1$ ...(ii)

From Eqs.(i) and (iii), $0 < p \le 1$

18 **(a)**

We have,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \left(\frac{\sin x^2}{x^2}\right) x = 1 \times 0 = 0 = f(0)$$

So, $f(x)$ is continuous at $x = 0$. $f(x)$ is also derivable at $x = 0$, because

$\lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \frac{\sin x^2}{x^2} = 1$ exists finitely

19 **(a)**

A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$ there exist $\delta > 0$, such that

$|f(x) - f(a)| < \in \quad \Rightarrow \quad |x - a| < \delta$

20 **(a)**

We have,

$f(x) = x - |x - x^2|, \qquad -1 \le x \le 1$

$$\Rightarrow f(x) = \begin{cases} x + x - x^2, & -1 \le x < 0 \\ x - (x - x^2), & 0 \le x \le 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2x - x^2, & -1 \le x < 0 \\ x^2, & 0 \le x \le 1 \end{cases}$$

Clearly, $f(x)$ is continuous at $x = 0$

Also,

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1} 2x - x^2 = -2 - 1 = -3 = f(-1)$

and,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1 = f(1)$

So, $f(x)$ is right continuous at $x = -1$ and left continuous at $x = 1$

Hence, $f(x)$ is continuous on $[-1, 1]$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	D	C	C	B	D	B	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	A	C	C	A	A	A	A	A