DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. : 1

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

We have,

$-\pi/4 < x < \pi/4$

$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \leq \tan^2 x < 1 \Rightarrow [\tan^2 x] = 0$

$\therefore f(x) = [\tan^2 x] = 0$ for all $x \in (-\pi/4, \pi/4)$

Thus, $f(x)$ is a constant function on $\in (-\pi/4, \pi/4)$

So, it is continuous on $\in (-\pi/4, \pi/4)$ and $f'(x) = 0$ for all $x \in (-\pi/4, \pi/4)$

2 **(d)**

Since, $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{-e^x + 2^x \log 2}{1} = f(0)$ [by L 'Hospital's rule]

$\Rightarrow f(0) = -1 + \log 2$

3 **(b)**

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$ for all x

$\Rightarrow -f'(-x) = f'(x)$ for all x

$\Rightarrow f'(-x) = -f'(x)$ for all x

$\Rightarrow f'(x)$ is an odd function

4 **(c)**

We have,

$f(x) = \begin{cases} [\cos \pi x], x < 1 \\ |x - 2|, 1 \leq x < 2 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 2 - x, & 1 \leq x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \leq 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \leq x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$

5 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0}\frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0}\frac{h}{\tan^{-1}(-h)} = \lim_{h\to 0}\frac{-h}{\tan^{-1}h} = -1$$

and,

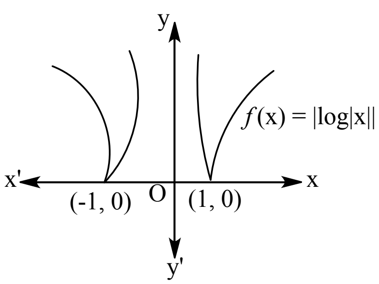
$$\lim_{x\to -2^+} f(x) = \lim_{h\to 0} f(-2+h) = \lim_{h\to 0}\frac{|-2+h+2|}{\tan^{-1}(-2+h+2)}$$

$$\Rightarrow \lim_{x\to -2^+} f(x) = \lim_{h\to 0}\frac{h}{\tan^{-1}h} = 1$$

$$\therefore \lim_{x\to -2^-} f(x) \neq \lim_{\to -2^+} f(x)$$

So, $f(x)$ is neither continuous nor differentiable at $x = -2$

6 **(b)**



From the graph of $f(x) = |\log|x||$ it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = \pm 1$, due to sharp edge

7 **(b)**

We have,

$$\lim_{x\to a}\frac{xf(a) - a\,f(x)}{x-a} = \lim_{x\to a}\frac{x\,f(a) - a\,f(a) - a(f(x) - f(a))}{x-a}$$

$$\Rightarrow \lim_{x\to a}\frac{x\,f(a) - a\,f(x)}{x-a} = \lim_{x\to a}\frac{f(a)(x-a)}{x-a} - a\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\Rightarrow \lim_{x\to a}\frac{x\,f(a) - a\,f(x)}{x-a} = f(a) - a\,f'(a) = 4 - 2a$$

8 **(c)**

Given, $f(x) = x\left(\sqrt{x} + \sqrt{x+1}\right)$. At $x = 0$ LHL of $\sqrt{x}$ is not defined, therefore it is not continuous at

$$x = 0$$

Hence, it is not differentiable at $x = 0$

9 **(a)**

Here, $f'(x) = \begin{cases} 2ax, & b \neq 0, x \leq 1 \\ 2bx + a, & x > 1 \end{cases}$

Since, $f(X)$ is continuous at $x = 1$

$$\therefore \lim_{h\to 0} f(x) = \lim_{h\to 1^+} f(x)$$

$$\Rightarrow a + b = b + a + c \Rightarrow c = 0$$

Also, $f(x)$ is differentiable at $x = 1$

$\therefore$ (LHD at $x = 1$)=(RHD at $x = 1$)

$\Rightarrow\ 2a = 2b(1) + a\ \Rightarrow\ a = 2b$

10 **(d)**

We have,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left\{\frac{x^2}{4} - \frac{3x}{4} + \frac{13}{4}\right\} = \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} |x - 3| = 2$$

and, $f(1) = |1 - 3| = 2$

$$\therefore \lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$$

So, $f(x)$ is continuous at $x = 1$

We have,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3} |x - 3| = 0,\ \lim_{x \to 3^+} f(x) = \lim_{x \to 3} |x - 3| = 0$$

and, $f(3) = 0$

$$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

So, $f(x)$ is continuous at $x = 3$

Now,

(LHD at $x = 1$)

$$= \left\{\frac{d}{dx}\left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\right)\right\}_{x=1} = \left\{\frac{x}{2} - \frac{3}{2}\right\}_{x=1} = \frac{1}{2} - \frac{3}{2} = -1$$

(RHD at $x = 1$) $= \left\{\frac{d}{dx}(-(x-3))\right\}_{x=1} = -1$

$\therefore$ (LHD at $x = 1$) = (RHD at $x = 1$)

So, $f(x)$ is differentiable at $x = 1$

11 **(d)**

$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Now, $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{2\sin x - \sin 2x}{2x\cos x}$ $\left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{x \to 0} \frac{2\cos x - 2\cos 2x}{2(\cos x - x\sin x)}$$

$$= \lim_{x \to 0} \frac{2 - 2}{2(1 - 0)} = 0$$

Since, $f(x)$ is continuous at $x = 0$

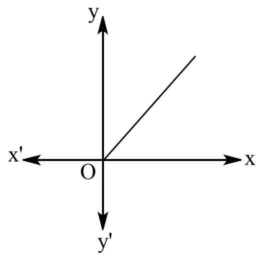
$$\therefore\quad f(0) = \lim_{x \to 0} f(x)$$

$\Rightarrow a = 0$

12 **(a)**

Given, $f(x) = x + |x|$

$$\therefore\quad f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph of $f(x)$ is continuous for every value of x

Alternate

Since, x and $|x|$ is continuous for every value of x, so their sum is also continous for every value of x

13 **(a)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

$\Rightarrow \lim_{x\to 0}\{1+|\sin x|\}^{\frac{a}{|\sin x|}} = b = \lim_{x\to 0} e^{\frac{\tan 2x}{\tan 3x}}$

$\Rightarrow e^a = b = e^{2/3} \Rightarrow a = \frac{2}{3}$ and $a = \log_e b$

14 **(b)**

We have,

$$f(x) = \begin{cases} x^2 + \dfrac{(x^2/1+x^2)}{1-(1/1+x^2)} = x^2+1, x \neq 0 \\ 0, \quad x = 0 \end{cases}$$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is discontinuous at $x = 0$

15 **(d)**

$\text{LHD} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0}\frac{1-1}{-h} = 0$

$\text{RHD} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{1+\sin(0+h)-1}{h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$

$\Rightarrow$ LHD $\neq$ RHD

16 **(a)**

Given, $f(x) = x - |x - x^2|$

At $x = 1$, $f(1) = 1 - |1-1| = 1$

$\lim_{x\to 1^{-1}} f(x) = \lim_{h\to 0}[(1-h) - |(1-h)-(1-h)^2|]$

$= \lim_{h\to 0}\left[(1-h) - |h-h^2|\right] = 1$

$\lim_{x\to 1^+} f(x) = \lim_{h\to 0}[(1+h) - |(1+h)-(1+h)^2|]$

$= \lim_{h\to 0}\left[1+h-|-h^2-h|\right] = 1$

$\because \lim_{x\to 1^{-1}} f(x) = \lim_{x\to 1^{+}} = f(1)$

17 **(a)**

We have,

$f(x+y+z) = f(x)f(y)f(z)$ for all x,y,z ...(i)

$\Rightarrow f(0) = f(0)f(0)f(0)$ [Putting $x = y = z = 0$]

$\Rightarrow f(0)\{1 - f(0)^2\} = 0$

$\Rightarrow f(0) = 1$ [$\because f(0) = 0 \Rightarrow f(x) = 0$ for all x]

Putting $z = 0$ and $y = 2$ in (i), we get

$f(x+2) = f(x)f(2)f(0)$

$\Rightarrow f(x+2) = 4f(x)$ for all x

$\Rightarrow f'(2) = 4f'(0)$ [Putting $x = 0$]

$\Rightarrow f'(2) = 4 \times 3 = 12$

18 **(b)**

For $x > 1$, we have

$$f(x) = |\log|x|| = \log x \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $x < -1$, we have

$$f(x) = |\log|x|| = \log(-x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $0 < x < 1$, we have

$$f(x) = |\log|x|| = -\log x \quad \Rightarrow \quad f'(x) = \frac{-1}{x}$$

For $-1 < x < 0$, we have

$$f(x) = -\log(-x) \quad \Rightarrow \quad f'(x) = -\frac{1}{x}$$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

19 **(c)**

Since, $\lim_{x\to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0} \frac{1-\cos x}{x^2} = k$$

$$\Rightarrow \lim_{x\to 0} \frac{-(-\sin x)}{2x} = k \quad \text{[using L 'Hospital's rule]}$$

$$\Rightarrow \frac{1}{2}\lim_{x\to 0} \frac{\sin x}{x} = k \quad \Rightarrow \quad k = \frac{1}{2}$$

20 **(b)**

Given, $f(X) = |x-1| + |x-2|$

$$= \begin{cases} x-1+x-2, & x \ge 2 \\ x-1+2-x, & 1 \le x < 2 \\ 1-x+2-x, & x < 1 \end{cases}$$

$$= \begin{cases} 2x-3, & x \ge 2 \\ 1, & 1 \le x < 2 \\ 3-2x, & x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x > 2 \\ 0, & 1 < x < 2 \\ -1, & x < 1 \end{cases}$$

Hence, except $x = 1$ and $x = 2$, $f(x)$ is differentiable everywhere in R

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	B	C	B	B	B	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	A	B	D	A	A	B	C	B