**DPP**
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

**Solutions**

Subject : Maths
DPP No. : 9

# Topic :-Applications of integrales

2 **(c)**

We have,

$$A=\int_0^{\pi/4}\sin x\,dx=[-\cos x]_0^{\pi/4}=\frac{\sqrt{2}-1}{\sqrt{2}}=1-\frac{1}{\sqrt{2}}\quad ...(i)$$

Let $A_1$ be the required area. Then,

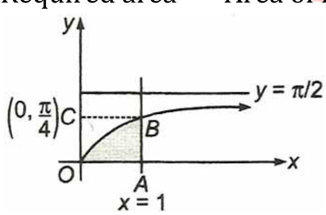
$$A_1=\int_0^{\pi/4}\cos x\,dx=[\sin x]_0^{\pi/4}=\frac{1}{\sqrt{2}}\quad ...(ii)$$

From (i) and (ii), we have

Required area $A_1=1-A$

3 **(b)**

Required area $=$ Area of rectangle $OABC-$ Area of curve $OABO$




$$\frac{\pi}{4}-\int_0^{\pi/4}\tan y\,dy$$

$$=\frac{\pi}{4}+[\log\cos y]_0^{\pi/4}$$

$$=\frac{\pi}{4}+\log\cos\frac{\pi}{4}-\log\cos(0)$$

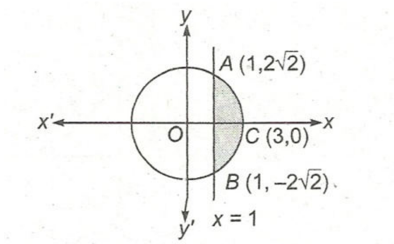
$$=\frac{\pi}{4}+\log 1-\log\sqrt{2}-\log 1$$

$$=\left(\frac{\pi}{4}-\log\sqrt{2}\right)\text{ sq unit}$$

4 **(b)**

Required area $=2\int_1^3\sqrt{9-x^2}\,dx$

$$= 2 \cdot \frac{1}{2}\left[x\sqrt{9 - x^2} + 9\sin^{-1}\frac{x}{3}\right]_1^3$$




$$= \left[9\sin^{-1}(1) - \sqrt{8} - 9\sin^{-1}\left(\frac{1}{3}\right)\right]$$

$$= \left[9\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\} - \sqrt{8}\right]\left[\because \cos^{-1}\theta = \frac{\pi}{2} - \sin^{-1}\theta\right]$$

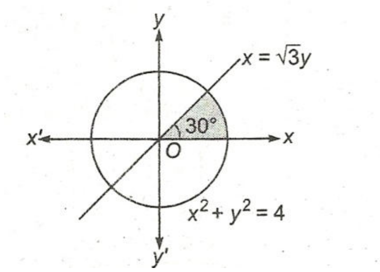
$$= [9\sec^{-1}(3) - \sqrt{8}]\text{sq unit}$$

5 **(c)**

Required area $= \int_0^1 (x_2 - x_1)dy$

$$= \int_0^1 \left(\sqrt{4 - y^2} - \sqrt{3}\, y\right) dy$$

$$= \left[\frac{1}{2}y\sqrt{4 - y^2} + \frac{1}{2}(4)\sin^{-1}\frac{y}{2} - \frac{\sqrt{3}\, y^2}{2}\right]_0^1$$




$$= \frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2} - 2\sin^{-1}0$$

$$= \frac{\pi}{3}\text{sq units}$$

**Alternate**

$$= \text{Area} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{30}{360} \times \pi(2)^2$$

$$= \frac{\pi}{3}\text{sq units}$$

8 **(d)**

Given equation of circle and line are

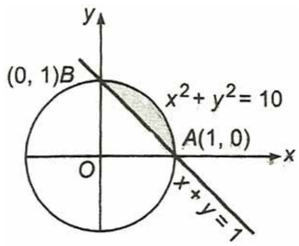
$x^2 + y^2 = 1$ ...(i)

and $x + y = 1$ ...(ii)

From Eqs. (i) and (ii),

$x^2 + (1 - x)^2 = 1$

$\Rightarrow x^2 + 1 + x^2 - 2x = 1$




$\Rightarrow 2x^2 - 2x = 0 \Rightarrow 2x(x - 1) = 0$

$\Rightarrow x = 0, x = 1 \Rightarrow y = 1, y = 0$

$\therefore$ Point of intersection of circle and line are $A(1, 0)$ and $B(0, 1)$

$\therefore$ Required area $= \int_0^1 \left[\sqrt{1 - x^2} - (1 - x)\right] dx$

$= \left[\frac{x\sqrt{1 - x^2}}{2} + \frac{1}{2}\sin^{-1} x - x + \frac{x^2}{2}\right]_0^1$

$= \frac{1}{2} \cdot \frac{\pi}{2} - 1 + \frac{1}{2}$

$= \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq unit

9 **(c)**

$\because \displaystyle\int_1^b f(x)dx = (b - 1)\sin(3b + 4)$

$\therefore$ On differentiating both sides with respect to $b$, we get

$f(b) = 3(b - 1)\cos(3b + 4) + \sin(3b + 4)$

$\therefore f(x) = 3(x - 1)\cos(3x + 4) + \sin(3x + 4)$

10 **(c)**

The required area $A$ is given by

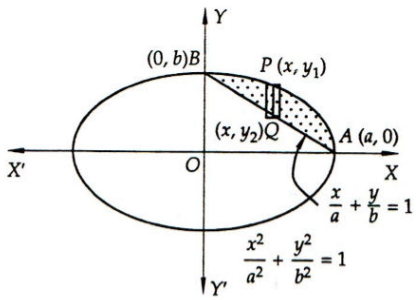
$$A = \int_0^a (y_1 - y_2)\, dx$$

$$\Rightarrow A = \int_0^a \left\{\frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a - x)\right\} dx$$

$$\Rightarrow A = \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \sin^{-1}\frac{x}{a}\right]^a + \frac{b}{2a}\left[(a - x)^2\right]_0^a$$

$$\Rightarrow A = \frac{b}{a}\left\{\frac{1}{2}a^2 \sin^{-1}(1)\right\} + \frac{b}{2a}(0 - a^2)$$

$$\Rightarrow A = \frac{\pi}{4}ab - \frac{1}{2}ab = \frac{ab}{4}(\pi - 2)$$



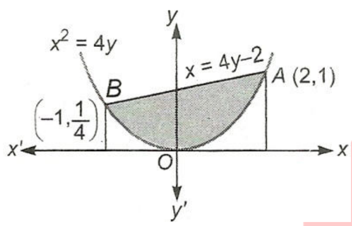

ALITER $A =$ Area of the ellipse in first quadrant – Area of $\Delta OAB$

$\Rightarrow A = \frac{\pi ab}{4} - \frac{1}{2}ab = \frac{ab}{4}(\pi - 2)$

11 **(b)**

The point of intersection of the parabola and the line are

$A(2,1)$and $B\left(1 - \frac{1}{4}\right)$



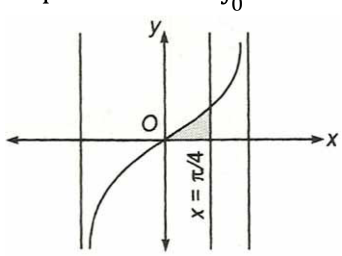

$\therefore$ The requerd area $= \left[\int_{-1}^{2} y\, dx\right] - \left[\int_{-1}^{2} y\, dx\right]$

$= \int_{-1}^{2} \frac{1}{4}(x+2)dx - \int_{-1}^{2} \frac{1}{4}\, x^2\, dx$

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2} = \frac{9}{8}$ sq units

12 **(d)**

Required area $= \int_0^{\pi/4} \tan x dx = [\, \log \sec x\, ]_0^{\pi/4}$




$= \log \sec\left(\frac{\pi}{4}\right) - \log \sec 0$

$= \log\sqrt{2} - \log 1 = \log\sqrt{2}$ sq unit

13 **(b)**

We have,

$A_1 = 2\int_0^a \sqrt{4ax}\, dx$ and, $A_2 = 2\int_0^{2a} \sqrt{4ax} - 2\int_0^a \sqrt{4ax}\, dx$

$\Rightarrow A_1 = \frac{8a^2}{3}$ and, $A_2 = \frac{16}{3}\sqrt{2}a^2 - \frac{8}{3}a^2$

$\Rightarrow \frac{A_1}{A_2} = \frac{1}{2\sqrt{2}-1} - \frac{2\sqrt{2}+1}{7}$

14 **(a)**

Required area

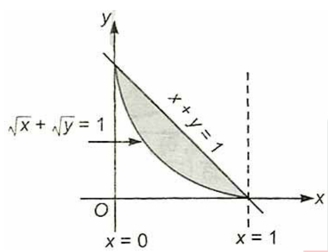
$$A = \int_{-1}^{1} (-x^2 + 2)dx + \int_{1}^{2} (2x - 1)dx$$

$$= \left[-\frac{x^3}{3} + 2x\right]_{-1}^{1} + [x^2 - x]_1^2$$

$$= \frac{10}{3} + 2 = \frac{16}{3} \text{ sq unit}$$

16 **(a)**

Required area $= \text{area of } \Delta\, AOB - \int_0^1 (1 - \sqrt{x})^2 dx$




$$= \frac{1}{2} \times 1 \times 1 - \left[x - \frac{2x^{3/2}}{3}\right]_0^1$$

$$= \frac{1}{3} \text{ sq unit}$$

17 **(c)**

Given area bounded by the curve, $y = \sqrt{3x+4}$,$x$-axis and the line $x = -1$ and $x = 4$ is $A$ and area bounded by the curve $y = \sqrt{3x+4}$

*ie*, $y = \pm (3x+4)^{1/2}$ $x$-axis and the line $x = -1$ and $x = 4$ is $B$

$\therefore B = 2A$

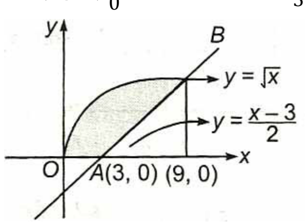
[Since, it is the area of both sides about x-axis]

Now, $A{:}B = A{:}2\,A = 1{:}2$

18 **(a)**

Required area $= \int_0^9 \sqrt{x}dx - \int_3^9 \left(\frac{x-3}{2}\right)dx$

$$= \left(\frac{x^{3/2}}{3/2}\right)_0^9 - \frac{1}{2}\left(\frac{x^2}{2} - 3x\right)_3^9$$



$$= \left(\frac{2}{7} \cdot 27\right) - \frac{1}{2}\left\{\left(\frac{81}{2} - 27\right) - \left(\frac{9}{2} - 9\right)\right\}$$

$= 18 - 9 = 9$ sq unit

20 **(a)**

Given, $\int_{\pi/4}^{\beta} f(x)dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta$

On differentiating w.r.t. β on both sides, we get

$f(\beta) = \sin\beta + \beta\cos\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$

Put $\beta = \frac{\pi}{2}$

Then, $f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} - \frac{\pi}{4}\sin\frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | C | B | B | C | B | C | D | C | C |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | B | D | B | A | B | A | C | A | B | A |
| | | | | | | | | | | |