DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 8

Topic :-Applications of integrales

1 **(b)**

Curved surface $= \int_a^b 2\ \pi y \sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]} dx$

Given that, $a = 2b = 3$ and $y = x + 1$

$\therefore \frac{dy}{dx} = 1 + 0 \Rightarrow \frac{dy}{dx} = 1$

Therefore, curved surface

$= \int_2^3 2\pi(x+1)\sqrt{[1+(1)^2]}dx = 2\sqrt{2}\ \pi \int_2^3 (x+1)dx$

$= 2\sqrt{2}\pi\left[\frac{(x+1)^2}{2}\right]_2^3 = \sqrt{2}\pi(16-9) = 7\pi\sqrt{2}$

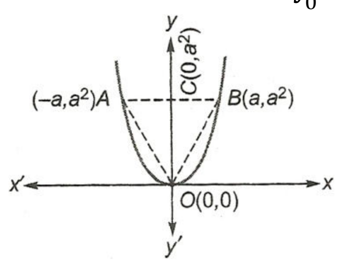
2 **(a)**

Required area $= 2\int_1^4 \sqrt{x}\ dx$

$= 2\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{4}{3}[8-1] = \frac{28}{3}$ sq units

3 **(b)**

Area of curve $OAB = 2\int_0^{a^2} x\ dy$



$= 2\int_0^{a^2} \sqrt{y}\ dy = 2\left[\frac{y^{3/2}}{3/2}\right]_0^{a^2}$

$= \frac{4}{3}[a^3]$

Now, Area of $\Delta OAB = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 2a \times a^2 = a^3$

$\therefore \frac{\text{Area of } \Delta\ AOB}{\text{Area of curve } AOB} = \frac{a^3}{\frac{4}{3}a^3} = \frac{3}{4}$

4 **(b)**

Area bounded by curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is given by

$A = \int_0^2 2^{kx} dx$

$= \left[\frac{2^{kx}}{k \log 2}\right]_0^2 = \left[\frac{2^{2k} - 1}{k \log 2}\right]$

But $A = \frac{3}{\log 2}$

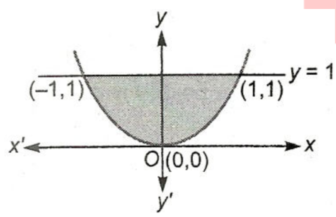
$\therefore \frac{2^{2k} - 1}{k \log 2} = \frac{3}{\log 2} \Rightarrow 2^{2k} - 1 = 3k$

This, relation is satisfied by only option (b)

7 **(b)**

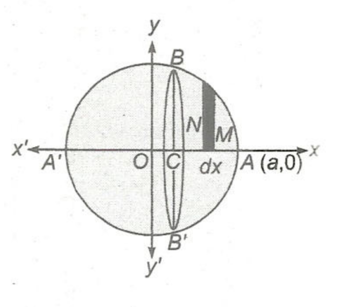
Required volume $= \pi \int_{-1}^{1} y^2 dx = 2\pi \int_0^1 x^4 dx$

$= 2\pi \left[\frac{x^5}{5}\right]_0^1 = \frac{2\pi}{5}$ cu unit



8 **(a)**

The required volume of the segment is generated by revolving the area $ABCA$ of the circle $x^2 + y^2 = a^2$ about the x-axis and for the arc BA.



Here, $CA = h$

and $OA = a$ [given]

$\therefore OC = OA - CA = a - h$

$\therefore x$ varies from $a - h$ to a

$\therefore$ The required volume $= \int_{a-h}^{a} \pi y^2 \, dx$

$= \pi \int_{a-h}^{a} (a^2 - x^2) dx = \pi \left[a^2 x - \frac{1}{3} x^3 \right]_{a-h}^{a}$

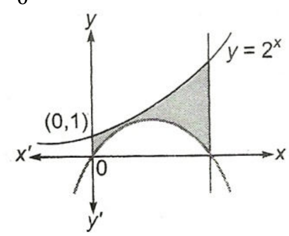
$= \pi \left[\left(a^3 - \frac{1}{3} a^3 \right) - \left\{ a^3 - a^2 h - \frac{1}{3} (a^3 - 3a^2 h + 3ah^2 - h^3) \right\} \right]$

$= \pi \left[a^2 h - a^2 h + ah^2 - \frac{1}{3} h^3 \right] = \frac{1}{3} \pi h^2 (3a - h)$

9 **(b)**

Required area

$= \int_0^2 [2^x - (2x - 2^2)] dx$



$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3} \right]_0^2$

$= \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2}$

$= \left(\frac{3}{\log 2} - \frac{4}{3} \right)$sq unit

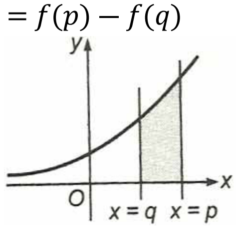
10 **(b)**

Required area $= \int_q^p ce^x dx$

$= [ce^x]_q^p$

$= c[e^p - e^q]$

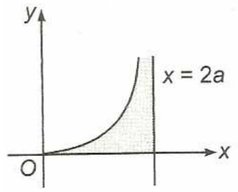
$= f(p) - f(q)$



12 **(b)**

Given equation of curve is

$y^2 (2a - x) = x^3$



Which is symmetrical about x-axis and passes through origin

Also, $\frac{x^3}{2a-x} < 0$

For $x > 2a$ or $x < 0$

So, curve does not lie in $x > 2a$ and $x < 0$, therefore curve lies wholly on $0 \leq x \leq 2a$

$\therefore$ Required area $= \int_0^{2a} \frac{x^{3/2}}{\sqrt{2a-x}} dx$

Put $x = 2\, a\sin^2\theta$

$\Rightarrow dx = 2a \cdot 2\sin\theta + \cos\theta d\theta$

$\therefore$ Required area $= \int_0^{\pi/2} 8a^2 \sin^4\theta d\theta$

$= 8a^2\left[\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right]$ (using gamma function)

$= \frac{3\pi a^2}{2}$ sq unit

13 **(c)**

Required area $= 2\int_0^a \sqrt{4ax}dx$

$= 4\sqrt{a} \times \frac{2}{3}[x^{3/2}]_0^a = \frac{8}{3}a^2$ sq unit

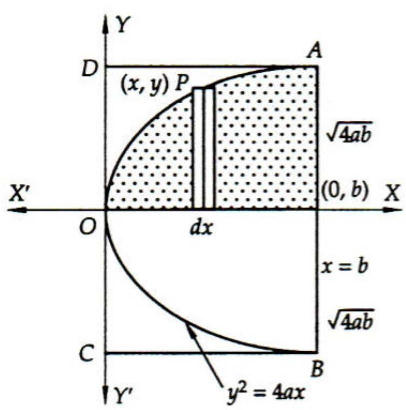
14 **(c)**

Let $y^2 = 4ax$ be a parabola and let $x = b$ be a double ordinate. Then,

$A_1 =$ Area enclosed by the parabola $y^2 = 4ax$ and the double ordinate $x = b$

$$\Rightarrow A_1 = 2\int_0^b y\, dx = 2\int_0^b \sqrt{4ax}\, dx = 4\sqrt{a}\int_0^b \sqrt{x^3}\, dx$$

$$\Rightarrow A_1 = 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^b = 4\sqrt{a} \times \frac{2}{3}b^{3/2} = \frac{8}{3}a^{1/2}b^{3/2}$$



And, $A_2 =$ Area of the rectangle $ABCD$

$\Rightarrow A_2 = AB \times AD = 2\sqrt{4ab} \times b = 4\, a^{1/2}b^{3/2}$

$\therefore A_1 : A_2 = 8/3\; a^{1/2}b^{3/2} : 4\; a^{1/2}b^{3/2} = 2/3 : 1 = 2 : 3$

16 **(b)**

The curve $y^2(2a-x)=x^3$ is symmetrical about x-axis and passes through origin.

Also, $\frac{x^3}{2a-x}<0\ for\ x>2a$ and $x<0$

So, curve does not lie in $x>2a$ and $x<0$, therefore curves lies wholly on $0\le x\le 2a$

$\therefore$ Requried area $=\int_0^{2a}\frac{x^{3/2}}{\sqrt{2a-x}}dx$

Put $x=2a\sin^2\theta$

$\Longrightarrow\ 0\ dx=4a\sin\theta\cos\theta\ d\ \theta$

$\therefore$ Requried area $=\int_0^{\frac{\pi}{2}}8a^2\sin^4\theta\ d\ \theta$

$=8a^2\left[\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right]$

$=\frac{3\pi a^2}{2}$ sq unit

17 **(b)**

Intersection points of given curves are $(-1,0)$ and $(3,0)$

Required area $=\int_{-1}^{3}(-x^2+2x+3)dx$

$=\left[\frac{x^3}{3}+\frac{2x^2}{2}+3x\right]_{-1}^{3}$

$=\left[-9+9+9-\left(\frac{1}{3}+1-3\right)\right]$

$=\frac{32}{3}$ sq units

18 **(b)**

Given curve $a^4y^2=(2a-x)x^5$

Cut off x-axis, when $y=0$

$0=(2a-x)x^5$

$\therefore x=0,2a$

Hence, the area bounded by the curve

$a^4y^2=(2a-x)x^5$ is

$A_1=\int_0^{2a}\frac{\sqrt{(2a-x)}x^{5/2}}{a^2}dx$

Put $x=2a\sin^2\theta$

$\therefore dx=4a\sin\theta\cos\theta d\theta$

$\therefore A_1=\int_0^{\pi/2}\frac{\sqrt{2a}\cos\theta(2a)^{5/2}\sin^5\theta\ 4a\sin\theta\cos\theta}{a^2}d\theta$

$=32a^2\int_0^{\pi/2}\sin^6\theta\cos^2\theta\ d\theta$

$=32a^2\cdot\frac{(5\cdot3\cdot1)(1)}{8\cdot6\cdot4\cdot2}\cdot\frac{\pi}{2}$ (by walli's formula)

$= \frac{5\pi a^2}{8}$

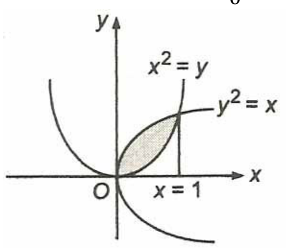
Area of circle, $A_2 = \pi a^2$

$\therefore \frac{A_1}{A_2} = \frac{5}{8}$

$\Rightarrow A_1 : A_2 = 5:8$

19 **(d)**

Required area $= \int_0^1 (\sqrt{x} - x^2) dx$



$= \left[\frac{2x^{3/2}}{3} - \frac{x^3}{3}\right]_0^1 = \left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{3}$ sq unit

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ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	B	A	B	B	A	B	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	C	C	C	B	B	B	D	C