# DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

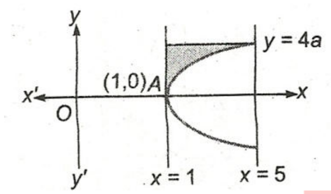
**Solutions**

Subject : Maths
DPP No. : 7

# Topic :-Applications of integrales

2 **(b)**

On solving $y^2 = 4a^2(x-1)$and $y = 4a$, we get $x = 5$



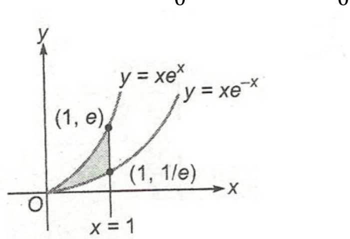

$\therefore$ Required area $= \int_1^5 (4a - 2a\sqrt{x-1})dx$

$= \left[4ax - 2a\frac{(x-1)^{3/2}}{3/2}\right]_1^5$

$= \frac{16a}{3}$ sq units

3 **(d)**

Required area $= \int_0^1 xe^x\ dx - \int_0^1 xe^{-x}\ dx$

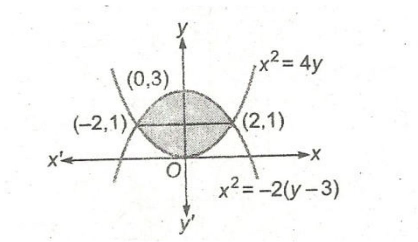



$= [xe^x - e^x]_0^1 - [-xe^{-x}e^{-x}]_0^1 = \frac{2}{e}$ sq unit

4 **(a)**

The point intersections of given curves are (2,1) and $(-2,1)$.

$\therefore$ Required area $= 2\int_0^1 x\,dy + 2\int_1^3 x\ dy$



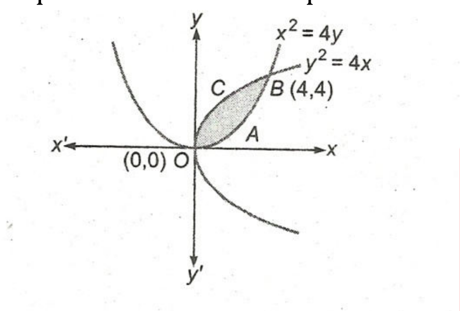

$= 2\int_0^1 \sqrt{4y}\ dy + 2\int_1^3 \sqrt{6-2y}\ dy$

$= 4\left[\frac{y^{3/2}}{3/2}\right]_0^1 + 2\left[\frac{(6-2y)^{3/2}}{3/2} \times \frac{1}{-2}\right]_1^3$

$= \frac{8}{3} + \frac{16}{3} = 8$ sq units

5 **(b)**

Required area of shaded portion $OABCO$




$= \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$

$= \left[\frac{2x^{3/2}}{3/2} - \frac{x^3}{12}\right]_0^4$

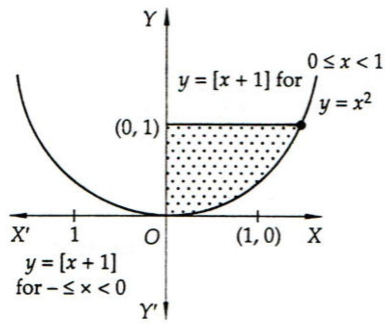
$= \left[\frac{32}{3} - \frac{16}{3}\right]$

$= \frac{16}{3}$ sq units

6 **(b)**

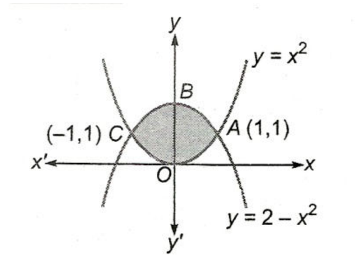
We have,

$\int_{\pi/3a}^{\pi/a} \sin ax\, dx = 3 \Rightarrow \frac{1}{a}\left[1 + \frac{1}{2}\right] = 3 \Rightarrow 2\, a = 1 \Rightarrow a = \frac{1}{2}$




7 **(a)**

Required area $= 2$ area of curve $OABO$




$= 2\int_0^1 [(2-x^2)-(x^2)]dx$

$= 2\int_0^1 (2-2x^2)dx$

$= 4\left[x - \frac{x^3}{3}\right]_0^1 = \frac{8}{3}$ sq units

9 **(a)**

$\because y = 2x^4 - x^2$

$\therefore \frac{dy}{dx} = 8x^3 - 2x$

For maxima or minima, put $\frac{dy}{dx} = 0$, we get

$x = -\frac{1}{2}, 0, \frac{1}{2}$

Then, $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} > 0$, $\left(\frac{d^2y}{dx^2}\right)_{x=0} < 0$

and $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} > 0$

$\therefore$ Required area $= \left|\int_{-1/2}^{1/2} (2x^4 - x^2)dx\right| = \frac{7}{120}$ sq unit

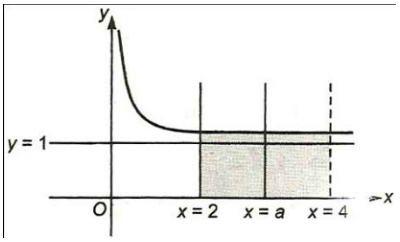
10 **(b)**

$\therefore$ Area $= \int_2^4 \left(1 + \frac{8}{x^2}\right)dx$

Since, the ordinate $x = a$ divides area into two equal parts, therefore,

$\int_2^a \left(1 + \frac{8}{x^2}\right)dx = \frac{1}{2}\int_2^4 \left(1 + \frac{8}{x^2}\right)dx$

$$\Rightarrow \left[x-\frac{8}{x}\right]_2^a = \frac{1}{2}\left[x-\frac{8}{x}\right]_2^4$$




$$\Rightarrow \left(a-\frac{8}{a}\right)-(2-4) = \frac{1}{2}[(4-2)-(2-4)]$$

$$\Rightarrow a-\frac{8}{a}+2 = 2$$

$$\Rightarrow a = \sqrt{8} = 2\sqrt{2} \text{ sq unit}$$

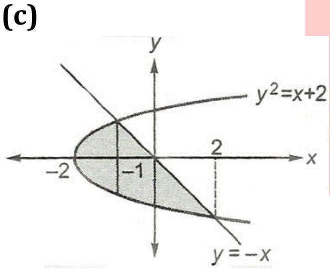
11 **(a)**

$$\text{Volume} = \int_0^1 \pi\ x^2\,dy - \int_0^1 \pi\ x^2\,dy$$

$$= \pi\int_0^1 9(1-x^2)dy - \pi\int_0^1 9(1-y)^2\,dy$$

$$= 9\pi\left[\left(y-\frac{y^3}{3}\right)+\frac{(1-y)^3}{3}\right]_0^1 = 9\pi\left[\left(1-\frac{1}{3}\right)+\left(0-\frac{1}{3}\right)\right] = 3\pi$$

12 **(c)**




Required area

$$= 2\int_{-2}^{-1}\sqrt{x+2}\,dx + \int_{-1}^{2}(-x+\sqrt{x+2})dx$$

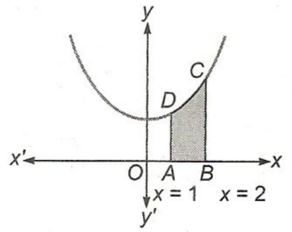
$$= \frac{4}{3}[(x+2)^{3/2}]_{-2}^{-1} + \left[\frac{-x^2}{2}+\frac{2}{3}(x+2)^{3/2}\right]_{-1}^{2}$$

$$= \frac{9}{2} \text{ sq units}$$

13 **(c)**

Required area = area of curve $ABCD$

$$= \int_1^2 (x^2+2)dx = \left[\frac{x^3}{3}+2x\right]_1^2$$




$= \left(\frac{8}{3} + 4\right) - \left(\frac{1}{3} + 2\right)$

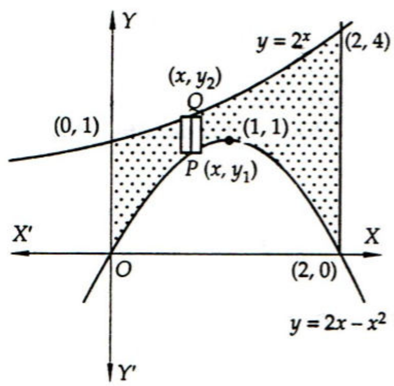
$= \frac{13}{3}$ sq units

14 **(d)**

Let the required area be $A$ sq. units. Then,

$A = \int_0^2 (y_2 - y_1)\, dx$

$\Rightarrow A = \int_0^2 \{2^x - (2x - x^2)\}\, dx$



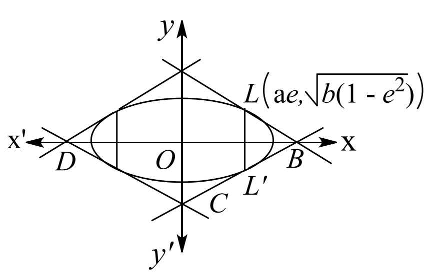

$\Rightarrow A = \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$

$\Rightarrow A = \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2}$

$\Rightarrow A = \frac{3}{\log 2} - \frac{4}{3}$

15 **(d)**

Given equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

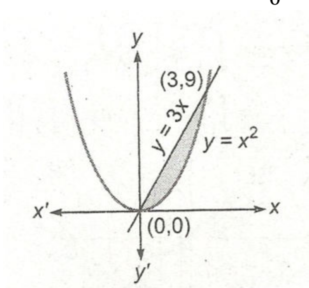


To find tangents at the end points of latusrectum we find $ae$,
$ie, ae = \sqrt{a^2 - b^2} = \sqrt{4} = 2$
By symmetry the quadrilateral is rhombus
So, area of rhombus is four times the area of the right angled formed by the tangent and axes in the first quadrant
$\Rightarrow$ Equation of tangent at $\left[ae, \sqrt{b(1-e^2)}\right] = \left(2, \frac{5}{3}\right)$ is
$$\frac{2}{9}x + \frac{5}{3} \cdot \frac{y}{5} = 1$$
$$\Rightarrow \frac{x}{9/2} + \frac{y}{3} = 1$$
$\therefore$ Area of quadrilateral $ABCD = 4(\text{area of } \Delta\, AOB)$
$= 4\left(\frac{1}{2} \cdot \frac{9}{2} \cdot 3\right) = 27$ sq unit

18 **(c)**

The intersection points of given curves are (0,0) and (3,9)

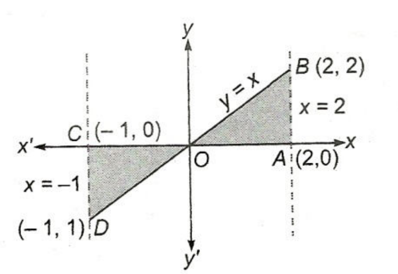
$\therefore$ Required area $= \int_0^3 (3x - x^2)dx$




$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{6} = 4.5 \text{ sq units}$$

20 **(a)**

Required area $= \int_{-1}^{2} y\,dx = \left|\int_{-1}^{0} y\,dx\right| + \int_0^2 y\,dx$




$$= \left|\int_{-1}^{0} x dx\right| + \int_0^2 x\,dx$$
$$= \left|\left[\frac{x^2}{2}\right]_{-1}^{0}\right| + \left[\frac{x^2}{2}\right]_0^2 = \frac{5}{2} \text{ sq unit}$$

**Alternate**

Required area $=$ Area of $\Delta OAB +$ Area of $\Delta OCD$

$\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 1$

$= \frac{5}{2}$ sq units

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | A | B | D | A | B | B | A | B | A | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | A | C | C | D | D | B | B | C | C | A |
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