

## Topic :- Applications of integrales

1 (a)

The given equation of curves are

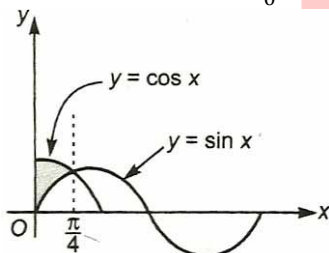
$$y = \sin x \quad \dots(i)$$

$$\text{and } y = \cos x \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$

$$\therefore \text{Required area} = \int_0^{\pi/4} (\cos x - \sin x) dx$$



$$= [\sin x + \cos x]_0^{\pi/4}$$

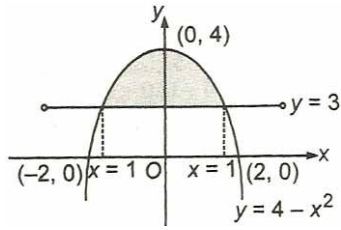
$$= \left( \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - \cos 0 \right)$$

$$= \left[ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right]$$

$$= \frac{2}{\sqrt{2}} - 1 = (\sqrt{2} - 1) \text{ sq unit}$$

4 (d)

$$y = \left[ 3 + \frac{x^2}{4} \right] = 3, -2 < x < 2$$



$$\text{Required area} = 2 \left\{ \int_0^1 (4 - x^2) dx - 3 \right\}$$

$$= 2 \left\{ \left[ 4x - \frac{x^3}{3} \right]_0^1 - 3 \right\}$$

$$= 2 \left\{ 4 - \frac{1}{3} - 3 \right\}$$

$$= \frac{4}{3} \text{ sq unit}$$

7 (d)

The given equation of curve can be written as  $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Here,  $a = 2$ ,  $b = 3$

$$\therefore \text{Required area} = \pi ab$$

$$= \pi \times 2 \times 3$$

$$= 6\pi \text{ sq units}$$

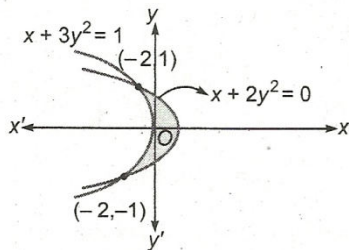
8 (a)

On solving the given curves, we get

$$y = \pm 1 \text{ and } x = -2$$

$$\therefore \text{Required area} = \left| \int_{-1}^1 (x_1 - x_2) dy \right|$$

$$= \left| \int_{-1}^1 (1 - 3y^2 + 2y^2) dy \right|$$



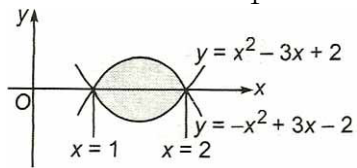
$$= \left| 2 \int_0^1 (1 - y^2) dy \right|$$

$$= \left| 2 \left[ y - \frac{y^3}{3} \right]_0^1 \right|$$

$$= \frac{4}{3} \text{ sq units}$$

9 (d)

Required area =  $2 \int_1^2 (-x^2 + 3x - 2) dx$  ( $\because$  both portions are same)



$$= 2 \left[ -\frac{x^3}{3} + \frac{3x^2}{2} - 2x \right]_1^2$$

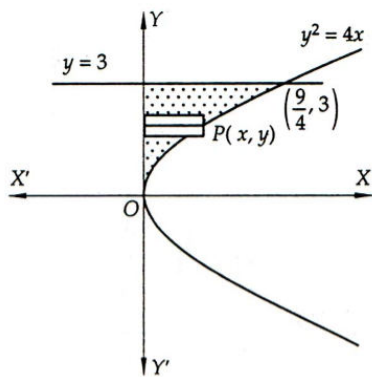
$$= 2 \left[ -\frac{8}{3} + 6 - 4 - \left( -\frac{1}{3} + \frac{3}{2} - 2 \right) \right]$$

$$= 2 \left[ -\frac{8}{3} + 2 + \frac{5}{6} \right] = \frac{1}{3} \text{ sq unit}$$

11 (b)

Let  $A$  be the required area. Then,

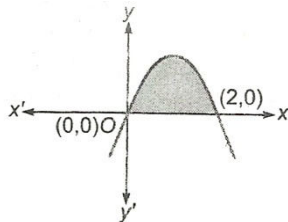
$$A = \int_0^3 x dy = \int_0^3 \frac{y^2}{4} dy = \left[ \frac{y^3}{12} \right]_0^3 = \frac{27}{12} = \frac{9}{4}$$



13 (b)

Given curve can be rewritten as

$$(x - 1)^2 = -(y - 1)$$



The curve cut the  $x$ -axis at  $(0, 0)$  and  $(2, 0)$

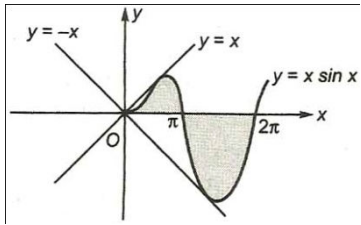
$$\therefore \text{Required area} = \int_0^2 (2x - x^2) dx$$

$$= \left[ x^2 - \frac{x^3}{3} \right]_0^2$$

$$= \frac{4}{3} \text{ sq units}$$

15 (c)

$$\text{Required area} = \int_0^{\pi} x \sin x \, dx + \left| \int_{\pi}^{2\pi} x \sin x \, dx \right|$$



$$\begin{aligned} &= \{-x \cos x + \sin x\}_0^{\pi} + \left| \{-x \cos x + \sin x\}_{\pi}^{2\pi} \right| \\ &= (\pi + 0) - (0 + 0) + |(-2\pi + 0) - (\pi + 0)| \\ &= \pi + 3\pi \\ &= 4\pi \text{ sq unit} \end{aligned}$$

PE

<b>ANSWER-KEY</b>										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	C	D	B	B	D	A	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	C	C	C	B	B	A	D

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