# DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

**Solutions**

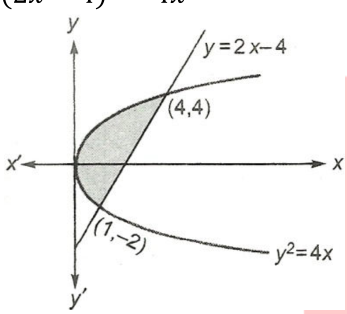
Subject : Maths
DPP No. : 4

## Topic :-Applications of integrales

1 **(c)**

The point of instruction of $y^2 = 4x$ and $y = 2x - 4$ is

$(2x-4)^2 = 4x$




$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-1)(x-4) = 0$

$\Rightarrow x = 1,4$

$\Rightarrow y = -2,4$

$\therefore$ Required area

$$\int_{-2}^{4}\left(\frac{y+4}{2}\right)dy - \int_{-2}^{4}\frac{y^2}{4}dy$$

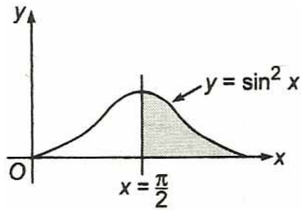
$$= \frac{1}{2}\left[\frac{y^2}{2} + 4y\right]_{-2}^{4} - \frac{1}{4}\left[\frac{y^3}{3}\right]_{-2}^{4}$$

$$= \frac{1}{2}[8 + 16 - (2-8)] - \frac{1}{12}[64 + 8]$$

$= 15 - 6 = 9$ sq units

3 **(b)**

Required area $A = \int_{\pi/2}^{\pi} \sin^2 x \, dx$




$= \frac{1}{2}\int_{\pi/2}^{\pi}(1-\cos 2x)dx$

$= \frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_{\pi/2}^{\pi}$

$= \frac{\pi}{4}$ sq unit

7 **(a)**

Given equation of curve is $y = a\sqrt{x} + bx$. This curve passes through (1, 2)

$\therefore 2 = a + b$ ...(i)

and area bounded by the curve and line $x = 4$ and $x$-axis is 8 sq unit, then

$\int_0^4 (a\sqrt{x} + bx)dx = 8$

$\Rightarrow \frac{2a}{3}[x^{3/2}]_0^4 + \frac{b}{2}[x^2]_0^4 = 8$

$\Rightarrow \frac{2a}{3}\cdot 8 + 8b = 8 \Rightarrow 2a + 3b = 3$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 3$ and $b = -1$

8 **(b)**

Given. area $= \int_0^2 2^{kx}\ dx = \frac{3}{\log 2}$

$\Rightarrow \left[\frac{2^{kx}}{\log_e 2}\right]_0^2 = \frac{3}{\log 2}$

$\Rightarrow \frac{2^{2k}}{\log_e 2} - \frac{1}{\log_e 2} = \frac{3}{\log 2}$

$\Rightarrow 2^{2k} - 1 = 3$

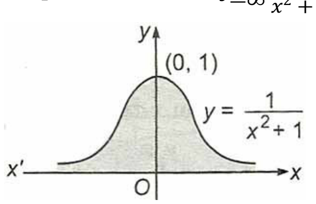
$\Rightarrow 2^{2k} = 2^2$

$\Rightarrow 2k = 2$

$\Rightarrow k = 1$

9 **(b)**

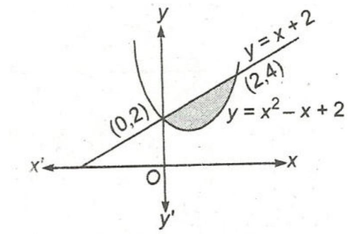
Required area $= \int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$




$= 2[\tan^{-1} x]_0^{\infty} = \pi$ sq unit

10 **(d)**

Given the equation of parabola can be rewritten as




$$\left(x-\frac{1}{2}\right)^2 = y-\frac{7}{4}$$

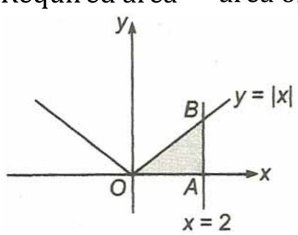
$$\therefore \text{ Required area} = \int_0^2 [(\mathrm{x}+2)-(x^2-x+2)]dx$$

$$\int_0^2 (-x^2+2x)dx$$

$$= \left[-\frac{x^3}{3}+x^2\right]_0^2 = -\frac{8}{3}+4 = \frac{4}{3} \text{ sq units}$$

11 **(c)**

Required area $=$ area of $\Delta OAB$

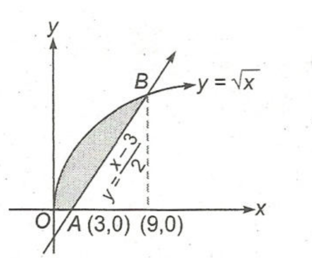



$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ sq unit}$$

12 **(a)**

Required area $OABO = \int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\frac{x-3}{2}\right) dx$

$$= \left(\frac{x^{3/2}}{3/2}\right)_0^9 - \frac{1}{2}\left(\frac{x^2}{2}-3x\right)_3^9$$




$$= \left(\frac{2}{3} \cdot 27\right) - \frac{1}{2}\left\{\left(\frac{81}{2}-27\right)-\left(\frac{9}{2}-9\right)\right\}$$

$= 9$ sq units

13 **(d)**

The given equation can be rewritten as

$$\frac{(x-\sqrt{2})^2}{2/\pi}+\frac{y^2}{8/\pi}=1$$

Which represent an ellipse.

Here, $a=\sqrt{\frac{2}{\pi}}$

and $b=\sqrt{\frac{8}{\pi}}$

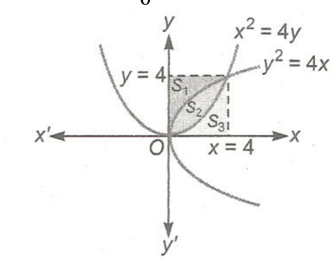
Area enclosed by an ellipse $=\pi ab$

$$=\pi\sqrt{\frac{2}{\pi}}\sqrt{\frac{8}{\pi}}$$

$=4$ sq units

15 **(a)**

$$S_1=S_3=\int_0^4\frac{x^2}{4}dx$$




$$=\frac{1}{4}\left[\frac{x^3}{3}\right]_0^4=\frac{16}{3}\text{ sq units}$$

Now, $S_2+S_3=\int_0^4\sqrt{4x}\,dx=2\times\left[\frac{x^{3/2}}{3/2}\right]_0^4=\frac{32}{3}$ sq units

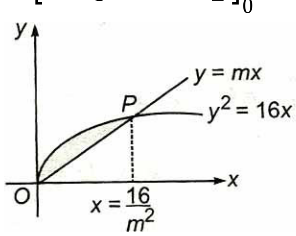
$\Rightarrow s_2=\frac{16}{3}$ sq units

$\therefore\ S_1{:}S_2{:}S_3=\frac{16}{3}:\frac{16}{3}:\frac{16}{3}=1{:}1{:}1$

16 **(b)**

Required area $=\int_0^{16/m^2}(\sqrt{16x}-mx)dx=\frac{2}{3}$ (given)

$$\Rightarrow\left[4\times\frac{2}{3}x^{\frac{3}{2}}-m\frac{x^2}{2}\right]_0^{16/m^2}=\frac{2}{3}$$



$$\Rightarrow \frac{8}{3} \times \frac{64}{m^3} - \frac{m}{2} \frac{256}{m^4} = \frac{2}{3}$$

$$\Rightarrow \frac{1}{m^3}\left[\frac{512}{3} - 128\right] = \frac{2}{3}$$

$$\Rightarrow m = 4$$

17 **(d)**

For $c < 1$, $\int_c^1 (8x^2 - x^5)dx = \frac{16}{3}$

$$\Rightarrow \frac{8}{3} - \frac{1}{6} - \frac{8c^3}{3} + \frac{c^6}{6} = \frac{16}{3}$$

$$\Rightarrow c^3\left[-\frac{8}{3} + \frac{c^3}{6}\right] = \frac{16}{3} - \frac{8}{3} + \frac{1}{6} = \frac{17}{6}$$

$\Rightarrow c = -1$ satisfy the above equation

For $c \geq 1$, none of the values of $c$ satisfy the required condition that

$$\int_1^c (8x^2 - x^5)dx = \frac{16}{3}$$

| **ANSWER-KEY** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | C | A | B | D | C | C | A | B | B | D |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | C | A | D | B | A | B | D | B | A | A |
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