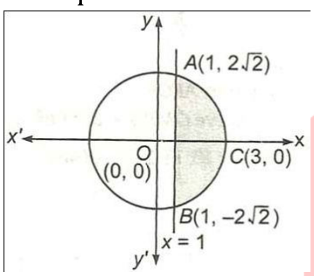
DPP
DAILY PRACTICE PRODUCTS

Class : XIth | **Solutions** | **Subject : Maths**
Date : | | **DPP No. : 3**

Topic :-Applications of integrales

1 **(b)**

The equation of circle is $x^2 + y^2 = 9$



∴ Area of the smaller segment cut off from the circle $x^2 + y^2 = 9$ by $x = 1$, given by

∴ Required area, $A = 2\int_1^3 \sqrt{9 - x^2}dx$

$= 2 \cdot \frac{1}{2}\left[x\sqrt{9 - x^2} + 9\sin^{-1}\frac{x}{3}\right]_1^3$

$= \left[\frac{9\pi}{2} - \sqrt{8} - 9\sin^{-1}\left(\frac{1}{3}\right)\right]$

$= \left[9\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)\right) - \sqrt{8}\right]$

$= \left[9\cos^{-1}\left(\frac{1}{3}\right) - \sqrt{8}\right]$

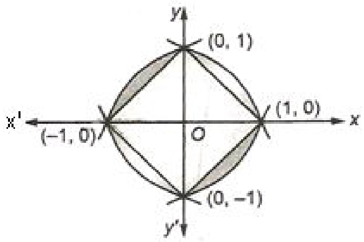
$= [9\sec^{-1}(3) - \sqrt{8}]$ sq unit

2 **(b)**

Since, $|x| + |y| = 1$

$\Rightarrow \begin{cases} x + y = 1, x > 0, y > 0 \\ x - y = 1, x > 0, y < 0 \\ -x + y = 1, x < 0, y > 0 \\ -x - y = 1, x < 0, y < 0 \end{cases}$

and $1 - y^2 = |x|$



$\Rightarrow \begin{cases} 1 - y^2 = x, x \geq 0 \\ 1 - y^2 = -x, x < 0 \end{cases}$

$\therefore$ Required area $= \left|2 \int_0^1 \sqrt{(1-x)}dx\right| + \left|2 \int_{-1}^0 \sqrt{(x+1)}dx\right| - 4\left(\frac{1}{2} \cdot 1 \cdot 1\right)$

$= \frac{2}{3}$ sq unit

5 **(c)**

Required area

$= 2 \int_{-1}^{3} \sqrt{x^2 - 1}\, dx$

$= 2\left[\frac{x\sqrt{x^2-1}}{2} - \frac{1}{2} \log\left|x + \sqrt{x^2-1}\right|\right]_{-1}^{3}$

$= \left(x\sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right|\right)_{-1}^{3}$

$= 6\sqrt{2} - \log\left|3 + 2\sqrt{2}\right|$

6 **(c)**

Requred area $= \int_1^2 \log x \; dx$

$= [x \log x - x]_1^2 = 2 \log 2 - 1$

$= \log 4 - \log e = \log\left(\frac{4}{e}\right)$

8 **(d)**

$\int_1^b f(x)dx = \sqrt{(b^2+1)} - \sqrt{2}$

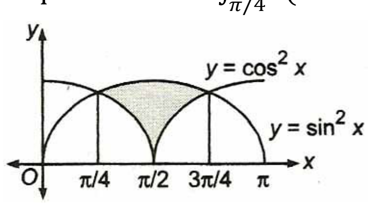
On differentiating both sides w.r.t.b, we get

$f(b) = \frac{b}{\sqrt{(b^2+1)}}$

Hence, $f(x) = \frac{x}{\sqrt{(x^2+1)}}$

9 **(c)**

Required area $= \int_{\pi/4}^{3\pi/4} (\sin^2 x - \cos^2 x)dx$



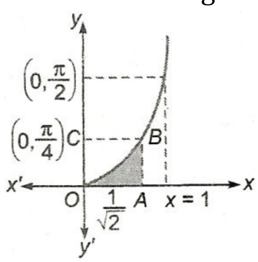
$$= -\int_{\pi/4}^{3\pi/4} \cos(2x)dx = -\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4}$$

$$= -\frac{1}{2}\left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right) = 1 \text{ sq unit}$$

10 **(d)**

Required area

= area of rectangle $OABC -$ area of curve $OBCO$



$$= \frac{\pi}{4\sqrt{2}} - \int_0^{\frac{\pi}{4}} \sin y \, dy$$

$$= \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - 1\right)\right] \text{ sq unit}$$

13 **(d)**

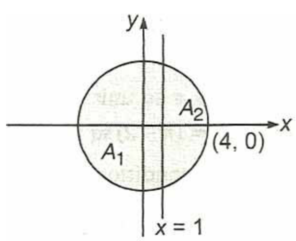
Required area $= \int_1^{1.7} [x]dx$

$$= \int_1^{1.7} dx = 1.7 - 1 = 0.7 = \frac{7}{10}$$

14 **(d)**

Equation of circle is $x^2 + y^2 = 16$

$\therefore$ Total area of circle $= A_1 + A_2 = 16\pi$ ...(i)



$\frac{A_1}{A_2} = \frac{16\pi}{A_2} - 1$ [on dividing Eq. (i) by A_2]

and $A_2 = 2\int_1^4 \sqrt{16 - x^2}dx$

$$A_2 = 2\left\{\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right\}_1^4$$

$$= 2\left\{4\pi - \frac{\sqrt{15}}{2} - 8\sin^{-1}\left(\frac{1}{4}\right)\right\}$$

$$= 8\pi - \sqrt{15} - 16\sin^{-1}\left(\frac{1}{4}\right)$$

$$\therefore \frac{A_1}{A_2} = \frac{16\pi}{8\pi - \sqrt{15} - 16\sin^{-1}\left(\frac{1}{4}\right)} - 1$$

15 **(b)**

Given equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Here, $a = 5, b = 4$

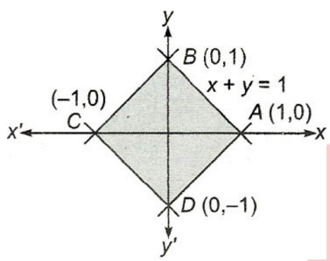
We know that the area of an ellipse $\frac{x^4}{a^2} + \frac{y^2}{b^2} = 1$ is

$\pi ab = \pi(5)(4) = 20\pi$ sq unit

17 **(d)**

$$\text{Area} = 4\int_0^1 (1-x)dx = 4\left[x - \frac{x^2}{2}\right]_0^1$$

$$= 4\left(1 - \frac{1}{2}\right) = 2 \text{ sq units}$$



Alternate

From figure $ABCD$ is square, whose diagonals AC and BD are of length 2 unit.

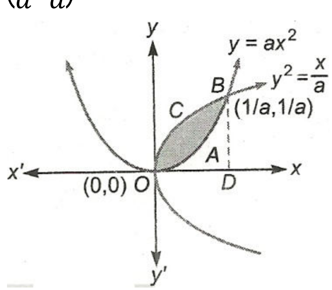
Hence, Required area $= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 2 \times 2$

$= 2$ sq units

19 **(a)**

The points of intersection of given curves are $(0,0)$ and $\left(\frac{1}{a}, \frac{1}{a}\right)$



$\therefore$ Required area $OABCO$ = area of $OCBDO$ − area of $OABDO$

$$\Rightarrow \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = 1 \quad \text{[given]}$$

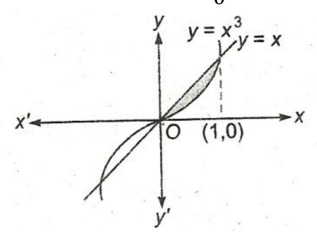
$$\Rightarrow \left(\frac{1}{\sqrt{a}} \cdot \frac{x^{3/2}}{3/2} - \frac{ax^3}{3}\right)_0^{1/a} = 1$$

$$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1$$

$$\Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \frac{1}{\sqrt{3}} \qquad [\text{as } a > 0]$$

20 **(a)**

Required area $= 2\int_0^1 (x - x^3)dx$



$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2} \text{ sq unit}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	C	C	C	D	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	D	D	B	C	D	B	A	A