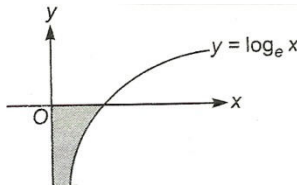


Topic :- Applications of integrales

2 (a)

Required area = $\left| \int_0^{-\infty} e^y dy \right|$

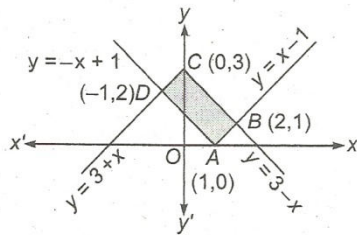


= $|[e^y]_0^{-\infty}| = 1$ sq unit

5 (c)

Given, $y = |x - 1| = \begin{cases} x - 1, & x > 1 \\ -x + 1, & x \leq 1 \end{cases}$

and $y = 3 - |x| = \begin{cases} 3 + x, & x \leq 0 \\ 3 - x, & x > 0 \end{cases}$



On solving $y = x - 1$ and $y = 3 - x$, we get $x = 2, y = 1$

Now, $AB^2 = (2 - 1)^2 + (1 - 0)^2 = 2$

$\Rightarrow AB = \sqrt{2}$

and $BC^2 = (0 - 2)^2 + (3 - 1)^2 = 8$

$\Rightarrow BC = 2\sqrt{2}$

\therefore Area of rectangle ABCD = $AB \times BC$

= $\sqrt{2} \times 2\sqrt{2} = 4$ sq units

6 (a)

Required area = $\int_1^2 (x^3 - x^2) dx$

$$= \left(\frac{x^4}{4} - \frac{x^3}{3} \right)_1$$

$$= \left(4 - \frac{8}{3} \right) - \left(\frac{1}{4} - \frac{1}{3} \right) = \frac{17}{22}$$

7 **(b)**

$$\text{Required area} = \int_{-2}^3 |x-3| dx$$

$$= \int_{-2}^{-1} |x-3| dx + \int_{-1}^0 |x-3| dx$$

$$+ \int_0^1 |x-3| dx + \int_1^2 |x-3| dx + \int_2^3 |x-3| dx$$

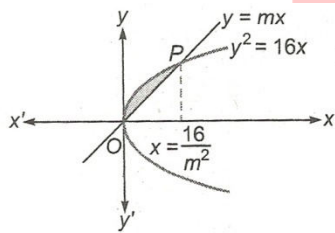
$$= \int_{-2}^{-1} 5 \cdot dx + \int_{-1}^0 4 \cdot dx + \int_0^{-1} 3 \cdot dx + \int_1^2 2 \cdot dx + \int_2^3 1 \cdot dx$$

$$= 5(1) + 4(1) + 3(1) + 2(1) + 1(1)$$

$$= 15 \text{ sq unit}$$

8 **(b)**

$$\text{Area} = \int_0^{16/m^2} (\sqrt{16x} - mx) dx = \frac{2}{3}$$



$$\Rightarrow \left[4 \cdot \frac{2}{3} x^{3/2} - \frac{mx^2}{2} \right]_0^{16/m^2} = \frac{2}{3}$$

$$\Rightarrow \frac{1}{m^3} \left[\frac{512}{3} - \frac{256}{2} \right] = \frac{2}{3}$$

$$\Rightarrow m^3 = \frac{128}{3} \times \frac{3}{2} = 64$$

$$\Rightarrow m = 4$$

9 **(d)**

$$\text{Required area} = \int_3^5 (3x-5) dx$$

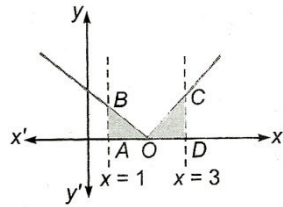
$$= \left(\frac{3x^2}{2} - 5x \right)_3^5 = \left(\frac{75}{2} - 25 \right) - \left(\frac{27}{2} - 15 \right)$$

$$= \frac{75}{2} - 25 - \frac{27}{2} + 15 = \frac{48}{2} - 10 = 14 \text{ sq units}$$

10 **(a)**

$$\text{Required area} = \int_1^3 |x-2| dx$$

$$= \int_1^2 (2-x)dx + \int_2^3 (x-2)dx$$



$$= \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^3$$

$$= 2 - \frac{3}{2} - \frac{3}{2} + 2 = 1 \text{ sq unit}$$

Alternate

Area = Area of ΔAOB + Area of ΔODC

$$= \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 1 \times 1$$

$$= 1 \text{ sq unit}$$

13 **(a)**

We have, $\frac{dy}{dx} = 2x + 1$

$\Rightarrow y = x^2 + x + c$, it passes through (1, 2)

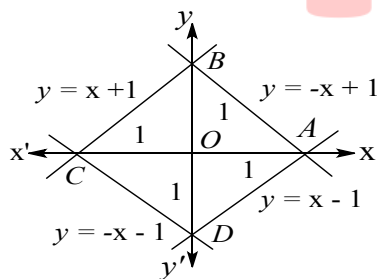
$$\therefore c = 0$$

Then, $y = x^2 + x$

$$\therefore \text{Required area} = \int_0^1 (x^2 + x)dx = \frac{5}{6} \text{ sq unit}$$

14 **(b)**

The lines are $y = x - 1, x \geq 0$



$y = -x - 1, x < 0 = -x + 1, x \geq 0$ and $y = x + 1, x < 0$

Required area = (4 \times area of ΔAOB)

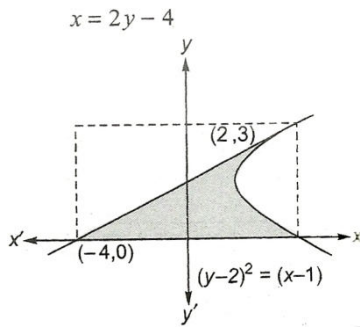
$$= 4 \times \left(\frac{1}{2} \times 1 \times 1 \right)$$

$$= 2 \text{ sq unit}$$

17 **(b)**

The equation of tangent at (2,3) to the given parabola is

$$x = 2y - 4$$



∴ Required area

$$\begin{aligned}
 &= \int_0^3 \{(y-2)^2 + 1 - 2y + 4\} dy \\
 &= \left[\frac{(y-2)^3}{3} - y^2 + 5y \right]_0^3 \\
 &= \frac{1}{3} - 9 + 15 + \frac{8}{3} \\
 &= 9 \text{ sq units}
 \end{aligned}$$

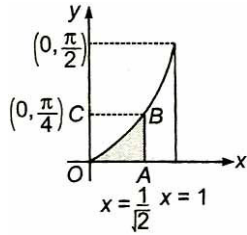
19 (a)

The equation of latusrectum of the parabola $y^2 = 12x$ is $x = 3$
Coordinates of end points of latusrectum are (3,6) and (3,-6)

$$\begin{aligned}
 \text{Required length} &= 2 \int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\
 &= 2 \int_0^3 \sqrt{1 + \left(\frac{6}{y}\right)^2} dx = 2 \int_0^3 \sqrt{\frac{12x + 36}{12x}} dx = 2 \int_0^3 \frac{x+3}{\sqrt{x^2+3x}} dx \\
 &= 2 \left[\int_0^3 \frac{2x+3}{2\sqrt{x^2+3x}} dx + \frac{3}{2} \int_0^3 \frac{1}{\sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}} dx \right] \\
 &= 2 \left[\sqrt{x^2+3x} + \frac{3}{2} \log \left| \left(x+\frac{3}{2}\right) + \sqrt{x^2+3x} \right| \right]_0^3 \\
 &= 2 \left[3\sqrt{2} + \frac{3}{2} \log \left(\frac{9}{2} + 3\sqrt{2} \right) - \frac{3}{2} \log \left(\frac{3}{2} \right) \right] \\
 &= 2 \left[3\sqrt{2} + 3 \log \left(3 + 2\sqrt{2} \right)^{\frac{1}{2}} \right] \\
 &= 2 \left[3\sqrt{2} + 3 \log (\sqrt{2} + 1) \right] \\
 &= 6 \left[\sqrt{2} + \log (1 + \sqrt{2}) \right]
 \end{aligned}$$

20 (d)

Required area



= Area of rectangle $OABC$ – Area of curve $OABO$

$$= \frac{\pi}{4\sqrt{2}} - \int_0^{\pi/4} \sin y \, dy$$

$$= \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\pi/4} = \left[\frac{\pi}{4\sqrt{2}} + \left\{ \frac{1}{\sqrt{2}} - 1 \right\} \right] \text{ sq unit}$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	B	C	C	A	A	B	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	A	B	D	D	B	D	A	D

PE