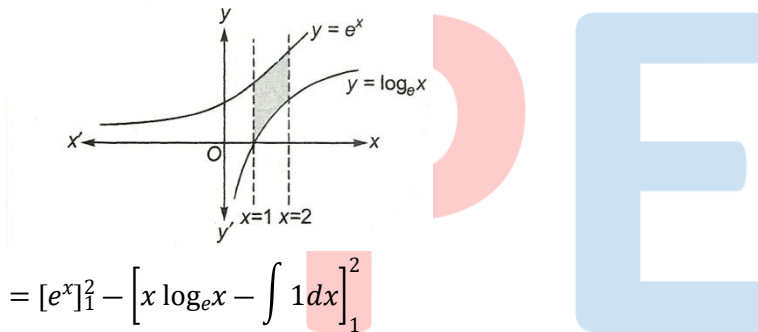


Topic :- Applications of integrales

2 (c)

Required area

$$A = \int_1^2 (e^x - \log_e x) dx$$



$$\begin{aligned} &= [e^x]_1^2 - \left[x \log_e x - \int_1^2 1 dx \right]_1^2 \\ &= e^2 - e - [x \log_e x - x]_1^2 \\ &= e^2 - e - [2 \log_e 2 - 2 - (0 - 1)] \\ &= e^2 - e - 2 \log_e 2 + 1 \end{aligned}$$

3 (b)

We have,

$$\int_1^k (8x^2 - x^5) dx = \frac{16}{3}$$

$$\Rightarrow \left[\frac{8x^3}{3} - \frac{x^6}{6} \right]_1^k = \frac{16}{3}$$

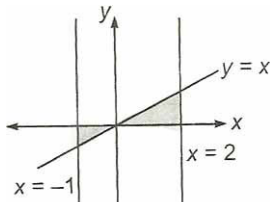
$$\Rightarrow \left(\frac{8k^3}{3} - \frac{k^6}{6} \right) - \left(\frac{8}{3} - \frac{1}{6} \right) = \frac{16}{3}$$

$$\Rightarrow 16k^3 - k^6 - 16 + 1 = 32$$

$$\Rightarrow k^6 - 16k^3 + 47 = 0 \Rightarrow k^3 = 8 \pm \sqrt{17} \Rightarrow k = (8 \pm \sqrt{17})^{1/3}$$

4 (d)

Required area



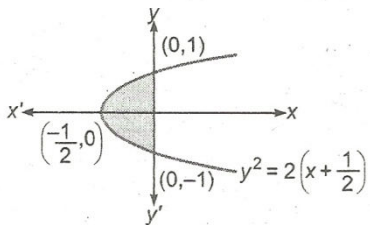
$$\begin{aligned} & \left| \int_{-1}^0 x \, dx \right| + \left| \int_0^2 x \, dx \right| \\ &= \left| \left[\frac{x^2}{2} \right]_{-1}^0 \right| + \left| \left[\frac{x^2}{2} \right]_0^2 \right| \\ &= \left| -\frac{1}{2} \right| + |2| \\ &= 2 + \frac{1}{2} = \frac{5}{2} \text{ sq unit} \end{aligned}$$

5

(b)

Given curve can be rewritten as

$$y^2 = 2\left(x + \frac{1}{2}\right)$$



$$\therefore \text{Required area} = \int_{-1}^1 x \, dy$$

$$= 2 \int_0^1 \frac{y^2 - 1}{2} \, dy$$

$$= \left| \left[\frac{y^3}{3} - y \right]_0^1 \right|$$

$$= \frac{2}{3} \text{ sq unit}$$

7

(a)

On solving the given equations of curves, we get $x = 0, 2$

$$\therefore \text{Required volume} = \pi \int_0^2 [(2x + 1)^2 - (x^2 + 1)^2] \, dx$$

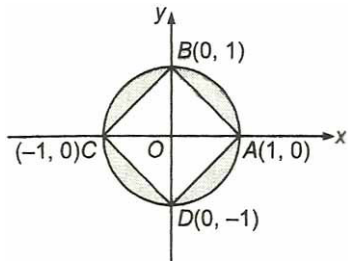
$$= \pi \int_0^2 (-x^4 + 2x^2 + 4x) \, dx$$

$$= \pi \left[-\frac{x^5}{5} + \frac{2x^3}{3} + \frac{4x^2}{2} \right]_0^2 = \frac{104\pi}{15} \text{ sq units}$$

8

(c)

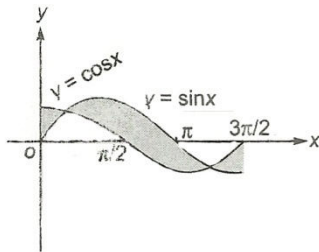
Area of square $ABCD = 2$ sq unit



Area of circle = π sq unit

\Rightarrow Required area = $(\pi - 2)$ sq unit

9 (a)



Required area

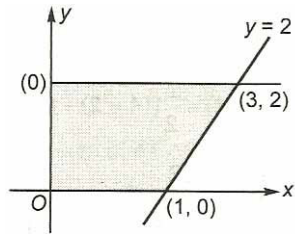
$$= \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) dx$$

$$= (\sin x + \cos x) \Big|_0^{\pi/4} + (-\cos x - \sin x) \Big|_{\pi/4}^{5\pi/4} + (\sin x + \cos x) \Big|_{5\pi/4}^{3\pi/2}$$

$$= (4\sqrt{2} - 2) \text{ sq units}$$

10 (c)

$$-8 < x < 8 \Rightarrow y = 2$$



$$\therefore \text{Required area} = \frac{1}{2}(1 + 3) \times 2$$

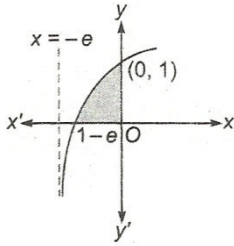
$$= 4 \text{ sq unit}$$

12 (d)

$$\text{Required area, } A = \int_{1-e}^0 \log_e(x + e) dx$$

$$\text{Put } x + e = t \Rightarrow dx = dt$$

$$\begin{aligned} \therefore A &= \int_1^e \log_e t \, dt \\ &= [t \log_e t - t]_1^e \\ &= (e - e - 0 + 1) \\ &= 1 \text{ sq unit} \end{aligned}$$



13 (a)

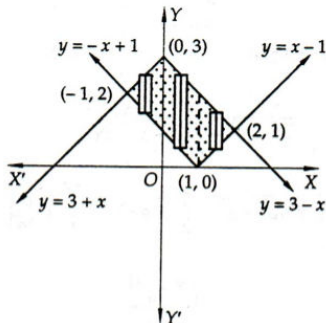
The two curves $y^2 = 4ax$ and $y = mx$ intersect at $(4a/m^2, 4a/m)$ and the area enclosed by the two curves is given by $\int_0^{4a/m^2} (\sqrt{4ax} - mx) dx$

$$\therefore \int_0^{4a/m^2} (\sqrt{4ax} - mx) dx = \frac{a^2}{3} \Rightarrow \frac{8a^2}{3m^3} = \frac{a^2}{3} \Rightarrow m^3 = 8 \Rightarrow m = 2$$

14 (c)

Let A be the required area. Then,

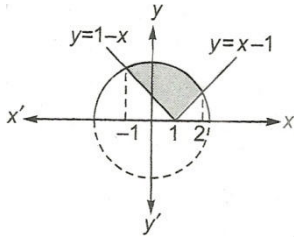
$$\begin{aligned} A &= \int_{-1}^0 \{(3+x) - (-x+1)\} \\ &+ \int_0^1 \{(3-x) - (-x+1)\} dx + \int_1^2 \{(3-x) - (x-1)\} dx \\ \Rightarrow A &= \int_{-1}^0 (2+2x) dx + \int_0^1 2 dx + \int_1^2 (4-2x) dx \\ \Rightarrow A &= [2x+x^2]_{-1}^0 + [2x]_0^1 + [4x-x^2]_1^2 = 4 \end{aligned}$$



15 (b)

Given, $y = \sqrt{5-x^2}$ and $y = |x-1|$
or $y^2 + x^2 = 5$

and $y = |x - 1|$



∴ Required area

$$\begin{aligned}
 &= \int_{-1}^2 \sqrt{5-x^2} dx - \int_{-1}^1 (1-x) dx - \int_1^2 (x-1) dx \\
 &= \left[\frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 - \left[x - \frac{x^2}{2} \right]_{-1}^1 - \left[\frac{x^2}{2} - x \right]_1^2 \\
 &= \left[1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} + 1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}} \right] - \left[1 - \frac{1}{2} - \left(-1 - \frac{1}{2} \right) \right] - \left[2 - 2 - \left(\frac{1}{2} - 1 \right) \right] \\
 &= 2 + \frac{5}{2} \sin^{-1} \left(\frac{2}{\sqrt{5}} \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{4}{5}} \right) - \frac{5}{2} \\
 &= \frac{5}{2} \sin^{-1}(1) = \frac{5\pi}{4} - \frac{1}{2} = \left(\frac{5\pi - 2}{4} \right) \text{sq unit}
 \end{aligned}$$

19 (a)

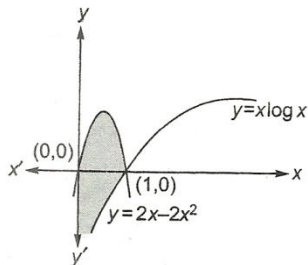
Volume of generated solid

$$\begin{aligned}
 &= \pi \int_0^2 x^2 dy = \pi \int_0^2 (9-y^2) dy = \pi \left[9y - \frac{1}{3}y^3 \right]_0^2 \\
 \pi &= \left[18 - \frac{8}{3} \right] = \frac{46}{3} \pi \text{ cu units}
 \end{aligned}$$

20 (c)

∴ Required area

$$= \int_0^1 [(2x - 2x^2) - (x \log x)] dx$$



$$\begin{aligned}
 &= \left[x^2 - \frac{2x^3}{3} - \left(\frac{x^2}{2} \log x - \frac{x^2}{4} \right) \right]_0^1 \\
 &= \left[1 - \frac{2}{3} - \left(0 - \frac{1}{4} \right) \right] = \frac{7}{12} \text{ sq unit}
 \end{aligned}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	B	D	B	C	A	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	A	C	B	B	A	C	A	C

PE