DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 6

Topic :-Applications of Derivatives

1 **(c)**

We have, $x = at^2 \Rightarrow \frac{dx}{dt} = 2at$

and $y = 2at \Rightarrow \frac{sy}{st} = 2a$

$\therefore$ Slope of tangent $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

$\Rightarrow \frac{1}{t} = \infty$

$\Rightarrow t = 0 \Rightarrow$ Point of contact is $(0, 0)$

2 **(d)**

Curve is $y^2 = px^3 + q$

$\therefore 2y\frac{dy}{dx} = 3px^2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{3p.4}{2.3}$

$\Rightarrow 4 = 2p$

$\Rightarrow p = 2$

Also, curve is passing through $(2, 3)$

$\therefore 9 = 8p + q$

$\Rightarrow q = -7$

$\therefore (p, q)$ is $(2, -7)$

3 **(b)**

Given, $x = t - 6t^2 + t^3$

On differentiating, w. r. t. x, we get

$\frac{dx}{dt} = 1 - 12t + 3t^2$

Again, differentiating, we get

$$\frac{d^2x}{dt^2} = -12 + 6t$$

When $\frac{d^2x}{dt^2} = 0 \Rightarrow t = 2$units time

4 **(a)**

Given equation of curve is

$y^2 = px^3 + q$

$\therefore \quad 2y\dfrac{dy}{dx} = 3px^2$

$\therefore \quad \left(\dfrac{dy}{dx}\right)_{(2,\,3)} = \dfrac{3p\,(2)^2}{2.3} = 2p$

The equation of tangent at (2, 3) is

$(y-3) = 2p(x-2)$

$\Rightarrow \quad y = 2px - (4p-3)$

This is similar to $y = 4x - 5$

$\therefore \quad 2p = 4$ and $4p - 3 = 5$

$\Rightarrow p = 2$ and $p = 2$

Since, the point (2, 3) lies on the curve

$\therefore \quad 9 = 8p + q \quad \Rightarrow \quad q = -7 \qquad [\because p = 2]$

5 **(c)**

Given, $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}, for\ u \in (-\infty, \infty)$

Now, $g(-u) = 2\tan^{-1}(e^{-u}) - \frac{\pi}{2}$

$= 2\left(\cot^{-1}(e^u)\right) - \dfrac{\pi}{2}$

$= 2\left(\dfrac{\pi}{2} - \tan^{-1}(e^u)\right) - \dfrac{\pi}{2}$

$= -g(u)$

$\therefore \quad g(u)$ Is an odd function.

Also, $g'(u) = 2\frac{1}{1+(e^u)^2}.e^u - 0 > 0$

Which is strictly increasing in $(-\infty, \infty)$

6 **(b)**

Given curve is $y = 2x^2 - x + 1$

Let the coordinate of P are (h, k)

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 4x - 1$

At the point (h, k), the slope

$= \left(\dfrac{dy}{dx}\right)_{(h,\,k)} = 4h - 1$

Since, the tangent is parallel to the given line $y = 3x + 4$

$\Rightarrow \; 4h - 1 = 3 \;\Rightarrow\; h = 1, k = 2$

$\therefore$ Coordinates of point P are (1, 2)

7 **(a)**

Let $f(x) = ax^2 + \frac{b}{x^2} - c$, where $a, b, c > 0$ and $x > 0$

Then, $f'(x) = 2ax - \frac{2b}{x^3}$ and $f''(x) = 2a + \frac{6b}{x^4}$

For local maximum or minimum, we must have

$f'(x) = 0 \Rightarrow x^4 = \dfrac{b}{a} \Rightarrow x = \pm\left(\dfrac{b}{a}\right)^{1/4}$

Clearly, $f''\left\{\pm\left(\frac{b}{a}\right)^{1/4}\right\} = 2a + 6a > 0$

Therefore, $x = \pm\left(\frac{b}{a}\right)^{1/4}$ are points of local minimum

Local minimum value of $f(x)$ is given by

$f\left\{\left(\frac{b}{a}\right)^{1/4}\right\} = a\left(\frac{b}{a}\right)^{1/2} + b\left(\frac{a}{b}\right)^{1/2} - c = 2\sqrt{ab} - c$

But, it is given that

$f(x) \geq 0$ for all x $\left[\because ax^2 + \frac{b}{x^2} \geq c\right]$

$\therefore 2\sqrt{ab} - c \geq 0 \Rightarrow 4ab \geq c^2$

8 **(a)**

Since $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum at $x = 1$

$\therefore f'(x) = 0$ at $x = 1$

$\Rightarrow f'(x) = 4x^3 - 124x + a = 0$ at $x = 1$

$\Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$

9 **(b)**

On differentiating given curve w.r.t. x, we get

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

The normal is parallel to x-axis, if

$-\left(\frac{dx}{dy}\right)_{(x_1, y_1)} = 0 \quad \Rightarrow \quad \sqrt{\frac{x_1}{y_1}} = 0 \Rightarrow \quad x_1 = 0$ ...(ii)

Since, the point (x_1, y_1) lies on a curve

$\therefore \quad \sqrt{x_1} + \sqrt{y_1} = \sqrt{a}$

$\Rightarrow \quad 0 + \sqrt{y_1} = \sqrt{a}$ [from Eq. (ii)]

$\Rightarrow \quad y_1 = a$

$\therefore$ Required point is $(0, a)$

10 **(a)**

$y = (2x - 1)e^{2(1-x)}$

$\Rightarrow \quad \frac{dy}{dx} = 2e^{2(1-x)} - 2(2x - 1)e^{2(1-x)}$

$= 2e^{2(1-x)}(2 - 2x)$

$= 4e^{2(1-x)}(1 - x)$

Put, $\frac{dy}{dx} = 0 \Rightarrow x = 1$

Now, $\frac{d^2y}{dx^2} = -8e^{2(1-x)}(1 - x) - 4e^{2(1-x)}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=1} = -4 < 0$

So, y is maximum at $x = 1$, when $x = 1, y = 1$.

Thus, the point of maximum is $(1,1)$.

The equation of the tangent at $(1,1)$ is

$y-1=0(x-1) \Rightarrow y=1$

11 **(c)**

If $f'''(a)>0$, then it is not an extreme point

12 **(b)**

Given function is $f(x)=x+\sin x$

On differentiating w.r.t. x, we get

$f'(x)=1+\cos x$

For maxima or minima put $f'(x)=0$

$\Rightarrow\ 1+\cos x\ =0\ \Rightarrow\ \cos x\ =-1\ \Rightarrow\ \ x=\pi$

Again differentiating w.r.t. x, we get

$f''(x)=-\sin x\,, at\ \ x=\pi f''(\pi)=0$

Again differentiating w.r.t. x, we get $f'''(x)=-\cos x$,

$f'''(\pi)=1$

At $x=\pi$, $f(x)$ is minimum

13 **(b)**

Let $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}4\pi r^2\frac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt}=\frac{30}{4\times\pi\times15\times15}=\frac{1}{30\pi}ft/\min\left[\because \frac{dV}{dt}=30, r=15\right]$$

14 **(a)**

Given, $\frac{x^2}{3}-\frac{y^2}{2}=1$ ...(i)

$\Rightarrow\ \frac{2x}{3}-y\frac{dy}{dx}=0$

$\Rightarrow\ \frac{dy}{dx}=\frac{2x}{3y}$

Now, slope of the line $y=x$ is 1

Since, tangent is parallel to given line, then

$\frac{2x}{3y}=1 \quad\Rightarrow\quad x=\frac{3y}{2}$ ...(ii)

From Eqs. (i) and (ii), we get $y=\pm 2$

$\therefore$ Points of contact are $(3,2)$ and $(-3,-2)$

$\therefore$ equations of tangents are

$y-2=(x-3) \quad\Rightarrow\quad x-y-1=0$

and $y+2=x+3 \quad\Rightarrow\quad x-y+1=0$

15 **(c)**

The two curves are

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i) and, $\frac{x^2}{l^2}-\frac{y^2}{m^2}=1$ ...(ii)

Differentiating with respect to x, we get

$\left(\frac{dy}{dx}\right)_{C_1}=-\frac{b^2}{a^2}\frac{x}{y}, \left(\frac{dy}{dx}\right)_{C_2}=\frac{m^2}{l^2}\frac{x}{y}$

The two curves intersect orthogonally, iff.

$$\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}=-1$$

$\Rightarrow -\frac{b^2}{a^2}\frac{x}{y}\times\frac{m^2}{l^2}\frac{x}{y}=-1$

$\Rightarrow m^2b^2x^2=a^2l^2y^2$ ...(iii)

Subtracting (ii) from (i), we get

$x^2\left(\frac{1}{a^2}-\frac{1}{l^2}\right)+y^2\left(\frac{1}{b^2}+\frac{1}{m^2}\right)=0$ ...(iv)

From (iii) and (iv), we get

$\frac{1}{m^2b^2}\left(\frac{1}{a^2}-\frac{1}{l^2}\right)=-\frac{1}{a^2l^2}\left(\frac{1}{b^2}+\frac{1}{m^2}\right)$

$\Rightarrow l^2-a^2=-b^2-m^2\Rightarrow a^2-b^2=l^2+m^2$

16 **(b)**

We have,

$\frac{dy}{dt}>\frac{dx}{dt},\frac{dy}{dt}>0$ and $\frac{dx}{dt}>0$

Now, $12y=x^3$

$\Rightarrow 12\frac{dy}{dt}=3x^2\frac{dx}{dt}\Rightarrow 3\left(x^2\frac{dx}{dt}-4\frac{dy}{dt}\right)=0\Rightarrow x^2\frac{dx}{dt}=4\frac{dy}{dt}$

We have,

$\frac{dy}{dt}>\frac{dx}{dt}$

$\Rightarrow \frac{x^2}{4}\frac{dx}{dt}>\frac{dx}{dt}$

$\Rightarrow (x^2-4)\frac{dx}{dt}>0\Rightarrow x^2-4>0\Rightarrow x\in(-\infty,-2)\cup(2,\infty)$

17 **(b)**

Let the centre of circle on y-axis be (0.k).

Let $d=\sqrt{(7-0)^2+(3-k)^2}$

$\Rightarrow \quad d^2=7^2+(3-k)^2=D$ (say)

On differentiating w. r. t. k , we get

$$\frac{dD}{dk}=0+2(3-k)(-1)$$

Put $\frac{dD}{dk}=0\Rightarrow k=3$

Now, $\frac{d^2D}{dk^2}=2>0$ *minima*,

$\therefore$ *Minimum* value at k=3 is d=7

Hence, minimum circle of radius 7.

18 **(b)**

$f(x)=e^x\sin x$

$f'(x)=e^x cosx+sin\,xe^x$

$f''(x)=-e^x sin\,x+\cos xe^x+e^x\cos x+e^x\sin x$

$2e^x cosx$

For maximum slope

$f''(x) = 0$

$\Rightarrow\ 2e^x \cos x\ = 0$

$\Rightarrow\quad \cos x = 0$

$\Rightarrow\ x = \frac{\pi}{2}, \frac{3\pi}{2}, \forall x \in [0,2\pi]$

$f'''(x) = 2\quad [-e^x \sin x + e^x \cos x]$

$at\ \ x = \frac{\pi}{2}, f'''(x) < 0$

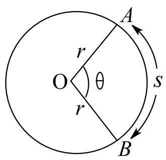
$and\ \ at\quad x = \frac{3\pi}{2} f'''(x) > 0$

$\therefore$ slop is maximum at $x = \frac{\pi}{2}$

19 **(d)**

$\because$ Perimeter of a sector $= p$

Let AOB be the sector with radius r



If angle of the sector be θ radius, then area of sector,

$A = \frac{1}{2} r^2 \theta$ ...(i)

And length of arc, $s = r\theta$

$\Rightarrow\quad \theta = \frac{s}{r}$

$\therefore$ perimeter of the sector

$p = r + s + r = 2r + s$ ...(ii)

On subtracting $\theta = \frac{s}{r}$ in Eq. (i), we get

$A = \left(\frac{1}{2} r^2\right)\left(\frac{s}{r}\right) = \frac{1}{2} rs$

$\Rightarrow\ s = \frac{2A}{r}$

Now, on substituting the value of s in Eq. (ii), we get

$p = 2r + \left(\frac{2A}{r}\right) \Rightarrow\ 2A = pr - 2r^2$

On differentiating w.r.t. r, we get

$2\frac{dA}{dr} = p - 4r$

For the maximum area, put

$2\frac{dA}{dr} = 0 \Rightarrow\ p - 4r = 0\ \Rightarrow\ r = \frac{p}{4}$

20 **(b)**

Given, $y^2 = x^3$

On differentiating w.r.t x, we get

$$2y\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{2y} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,\,1)} = \frac{3}{2}$$

$\therefore$ Equation of normal at point (1, 1) is

$$y - 1 = -\frac{2}{3}(x - 1)$$

$$\Rightarrow \quad 2x + 3y = 5$$

Hence, the equation of line parallel to above line will be in option (b), *ie*, $2x + 3y = 7$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	B	A	C	B	A	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	A	C	B	B	B	D	B