DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 5

Topic :-Applications of Derivatives

1 **(c)**

$p(t) = 1000 + \frac{1000t}{100 + t^2}$

$\Rightarrow \quad p'(t) = 0 + \frac{(100 + t^2)(1000) - 1000t\,(2t)}{(100 + t^2)^2}$

$= 1000\frac{(100 - t^2)}{(100 + t^2)^2}$

Put $p'(t) = 0$ for maxima or minima

$\Rightarrow \quad 100 - t^2 = 0$

$\Rightarrow \quad t = \pm 10$

Now, $p''(t) = 1000$

$\times \left[\frac{(100 + t^2)^2(-2t) - (100 - t^2)2(100 + t^2)2t}{(100 + t^2)^4}\right]$

$= 1000t\frac{[(100 + t^2)(-2) - (100 - t^2)(4)]}{(100 + t^2)^3}$

$= -1000t\,\frac{[600 - 2t^2]}{(100 + t^2)^3}$

At $t = 10,\ p''(t) < 0$

$\therefore$ The maximum value is

$p(10) = 1000 + \frac{10000}{100 + 100}$

$= 1000 + \frac{10000}{200} = 1050$

2 **(a)**

We have,

$f'(x) = (x - a)^{2n}(x - b)^{2m+1}$

$\therefore f'(x) = 0 \Rightarrow x = a, b$

For $x = b - h$, we have

$f'(x) = (b - h - a)^{2n}(-h)^{2m+1} < 0$

and for $x = b + h$, we have

$f'(x) = (b + h - a)^{2n}\,h^{2m+1} > 0$

Thus, as x passes through b, $f'(x)$ changes sign from negative
Hence, $x = b$ is a point of minimum

3 **(d)**

Given equation of curve is

$$y = 4 - 2x^2$$

$$\Rightarrow \frac{dy}{dt} = -4x\frac{dx}{dt}$$

Given $\frac{dx}{dt} = -5$, at point $(1,2)$

$$\therefore \frac{dy}{dt} = -4(1)(-5) = 20 \text{unit/s}$$

4 **(c)**

Given $y^2 = 2(x-3)$ ...(i)

$$\Rightarrow 2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}$$

Slope of the normal $= \frac{-1}{(dy/dx)} = -y$

Slope of the given line=2

$\therefore y = -2$

From Eq. (i), $x = 5$

$\therefore$ Required point is $(5, -2)$

5 **(a)**

Given, $f(x) = \frac{x}{1+|x|}$

$$\therefore f'(x) = \frac{(1+|x|).1 - x.\frac{|x|}{x}}{(1+|x|)^2}$$

$$= \frac{1}{(1+|x|)^2} > 0 \forall x \in R$$

$\Rightarrow f(x)$ is strictly increasing

6 **(d)**

Given, $f(x) = 2x^2 - 3x^2 + 90x + 174$

$\therefore f'(x) 6x^2 - 6x + 90$

Now, $D = b^2 - 4ac = 36 - 4 \times 6 \times 90 < 0$

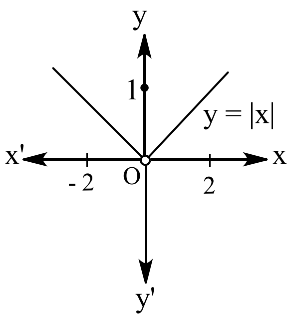
$\therefore f'(x) > 0 \forall x \in (-\infty, \infty)$

7 **(a)**

Given,

$$f(x) = \begin{cases} |x|, & \text{for } 0 < |x| \le 2 \\ 1, & \text{for } x = 0 \end{cases}$$

It is clear from the graph that $f(x)$ has local maximum.



8 **(a)**

We have,

$f(x) = x^2 + ax + 1$

$\Rightarrow f'(x) = 2x + a$

For $f(x)$ to be increasing on [1, 2], we must have

$f'(x) > 0$ for all $x \in R$

Now,

$f'(x) = 2x + a$

$\Rightarrow f''(x) = 2 > 0$ for all $x \in R$

$\Rightarrow f'(x)$ is increasing for all $x \in R$

$\Rightarrow f'(x)$ is increasing on [1, 2]

$\Rightarrow f'(1)$ is the minimum value of $f(x)$ in [1, 2]

Thus,

$f'(x) > 0$ for all $x \in [1, 2]$

$\Rightarrow f'(1) > 0$

$\Rightarrow 2 + a > 0 \Rightarrow a > -2 \Rightarrow a \in (-2, \infty)$

9 **(c)**

Since, $\frac{dx}{dt} = \frac{dy}{dt}$ ... (i)

Given equation of curve is

$y = x^2 + 2x$

$\Rightarrow \frac{dy}{dt} = (2x + 2)\frac{dx}{dt}$

$\Rightarrow 1 = 2x + 2$ [from Eq.(i)]

$\Rightarrow x = -1/2, \quad y = -3/4$

$\therefore$ point on the curve is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

10 **(b)**

Given, $p(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow p'(x) = 4x^3 + 3ax^2 + 2bx + c$

$\therefore$ $x = 0$ *is* a solution for p'(x)=0,

$\Rightarrow c = 0$

$\therefore$ $p(x) = x^4 + ax^3 + bx^2 + d$ ...(i)

Also , we have p(−1)<p(1)

$\Rightarrow 1 - a + b + d < 1 + a + b + d$

$\Rightarrow a > 0$

$\therefore$ $p'(x) = 0$, *only* when x=0 *and* $p(x)$ *is differentiable in* $(-1,1)$

, *we should have the maximum and minimum*

at the

points $x = -1,0$ and 1 only

Also ,we have $p(-1) < p(1)$

$\therefore$ Maximum of $p(x) =$ max{$p(0)$,$p(1)$}

And minimum of P(x)=Min {P(-1), P(0)}
In the interval [0,1]
$p'(x) = 4x^3 + 3ax^2 + 2bx$
$= x(4x^2 + 3ax + 2b)$
$\therefore p'(x) has\ only\ one\ root x = 0, then\ 4x^2 + 3ax + 2b = 0\ has$
No real roots.
$\therefore \quad (3a)^2 - 32b < 0$
$\Rightarrow \frac{3a^2}{32} < b$
$\therefore \quad b > 0$
Thus , we have a>0 and b>0
$\therefore \quad p'(x) = 4x^3 + 4ax^2 + 2bx > 0, \forall x \in (0,1)$
Hence ,$p(x) = p(1)$
Similarly,$p(x)$ is decreasing in [-1,0].
Therefore, Minimum $p(x)$does not occur at $x = -1$.

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$$p(t) = 1000 + \frac{1000t}{100 + t^2}$$

$$\Rightarrow \quad p'(t) = 0 + \frac{(100 + t^2)(1000) - 1000t\,(2t)}{(100 + t^2)^2}$$

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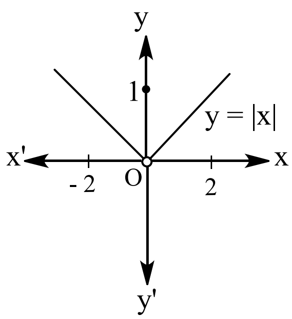

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ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	D	C	A	D	A	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	A	B	B	A	A	B	B	A