DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 4

Topic :-Applications of Derivatives

1 **(a)**

Let $y = x^3 - 12x \quad \Rightarrow \frac{dy}{dx} = 3x^2 - 12$

Put $\frac{dy}{dx} = 0, \quad 3x^2 - 12 = 0$

$\Rightarrow \quad x = \pm 2$

At $x = 2, \quad y = 2^3 - 12(2) = -16$

At $x = -2, \quad y = (-2)^3 - 12(-2) = 16$

Hence, option (a) is correct

2 **(c)**

We have,

$xy = a^2$ and $S = b^2x + c^2y$

$\Rightarrow S = b^2x + \frac{c^2a^2}{x}$

$\Rightarrow \frac{dS}{dx} = b^2 - \frac{c^2a^2}{x^2}$ and $\frac{d^3S}{dx^2} = \frac{2c^2a^2}{x^3}$

For local maximum or minimum, we must have

$\frac{dS}{x} = 0 \Rightarrow b^2 - \frac{c^2a^2}{x^2} = 0 \Rightarrow x^2 = \frac{c^2a^2}{b^2} \Rightarrow x = \pm \frac{ca}{b}$

Clearly, $\frac{d^2S}{dx^2} > 0$ for $x = \frac{ca}{b}$

So, $x = \frac{ca}{b}$ is the point of local minimum

Local minimum value of $S = b^2\left(\frac{ca}{b}\right) + c^2\left(\frac{a^2b}{ca}\right) = 2abc$

3 **(b)**

We have,

$g(x) = f(x) + f(1-x)$

$\therefore g'(x) = f'(x) - f'(1-x)$ for all $x \in [0,1]$

Now, $f"(x) < 0$ for $0 \leq x \leq 1$

$\Rightarrow f'(x)$ is a decreasing function on $[0, 1]$

$\Rightarrow f'(x) > f'(1 - x)$ if $x < 1 - x$

and,

$f'(x) < f'(1 - x)$ if $x > 1 - x$

$\Rightarrow f'(x) - f'(1 - x) > 0$ if $x < \frac{1}{2}$

and,

$f'(x) - f'(1 - x) < 0$ if $x > \frac{1}{2}$

$\Rightarrow g'(x) > 0$ if $x \in (0, 1/2)$

and,

$\Rightarrow g'(x) < 0$ if $x \in (1/2,1)$

$\Rightarrow g(x)$ decreases on $\left[\frac{1}{2,1}\right]$ and increases on $\left[\frac{0,1}{2}\right]$

4 **(d)**

Since, $f(x) = x\,e^{1-x}$

$f'(x) = -xe^{1-x} + e^{1-x}$

$= e^{1-x}(1-x)$

$\Rightarrow\ f'(x) < 0,\ \forall\ (1, \infty)$

5 **(a)**

Consider the function

$$\phi(x) = a_0\frac{x^{n+1}}{n+1} + a_1\frac{x^n}{n} + a_2\frac{x^{n-1}}{n-1} + \dots + a_{n-1}\frac{x^2}{2} + a_n x$$

Since $\phi(x)$ is a polynomial. Therefore, it is continuous on [0, 1] and differentiable on (0, 1)

Also, $\phi(0) = 0$

and, $\phi(1) = \dfrac{a_0}{n+1} + \dfrac{a_1}{n} + \dfrac{a_2}{n-2} + \dots + a_n = 0$ [Given]

$\therefore \phi(0) = \phi(1)$

Thus, $\phi(x)$ satisfies conditions of Rolle's theorem on [0, 1]

Consequently, there exist $c \in (0, 1)$ such that $\phi'(c) = 0$ i.e. $c \in (0, 1)$ is a zero of $\phi'(x) = a_0$ $x^n + a_1x^{n-1} + \dots + a_n = f(x)$

6 **(c)**

Given, $x = at^2 + bt + c$

$\Rightarrow (speed)\dfrac{dx}{dt} = 2at + b$

$\Rightarrow$ (acceleration)$\dfrac{d^2x}{dt^2} = 2a$

$\therefore$ The particle will moving with
Uniform acceleration.

9 **(a)**

On differentiating given equation w. r. t. x, we get

$\frac{dx}{dt} = 100 - \frac{25}{2}.(2t) = 100 - 25t$

At maximum height, velocity $\frac{dx}{dt} = 0$

$\therefore\quad 100 - 25t = 0 \ \Rightarrow\ t = 4$

$$\therefore \quad x = 100 \times 4 - \frac{25 \times 16}{2} = 200m$$

10 **(b)**

We have,

$y = \int_0^x |t| dt \quad ...(i)$

$\Rightarrow \frac{dy}{dx} = |x|$

Let $P(x_1, y_1)$ be a point on the curve (i) such that the tangent at P is parallel to the line

$$y = 2x$$

$\therefore$ (Slope of the tangent at P) = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 2 \Rightarrow |x_1| = 2 \Rightarrow x_1 = \pm 2$

Now, $y = \int_0^x |t| dt$

$$y = \begin{cases} \int_0^x t dt = \frac{x^2}{2}, & \text{if } x \geq 0 \\ -\int_0^x t\, dt = -\frac{x^2}{2}, & \text{if } x < 0 \end{cases}$$

$\therefore x_1 = 2 \Rightarrow y_1 = 2$ and $x_1 = -2 \Rightarrow y_1 = -2$

Thus, the two points on the curve are (2, 2) and $(-2, -2)$

The equations of the tangents at these two points are

$y - 2 = 2(x - 2)$ and $y + 2 = 2(x + 2)$ respectively

Or, $2x - y - 2 = 0$ and $2x - y + 2 = 0$ respectively

These tangents cut off intercepts -2 and 2 respectively on y-axis

11 **(a)**

We have,

$f'(x) = \sec^2 x - 1 \geq 0$ for all x $[\because |\sec x| \geq 1$ for all $x]$

Hence, $f(x)$ always increases

12 **(b)**

Let $P = xy$. Then,

$P = x(8 - x) \qquad [\because x + y = 8 \text{ (given)}]$

$\Rightarrow P = 8x - x^2 \Rightarrow \frac{dP}{dx} = 8 - 2x$ and $\frac{d^2P}{dx^2} = -2$

For maximum and minimum, we must have

$\frac{dP}{dx} = 0 \Rightarrow 8 - 2x = 0 \Rightarrow x = 4$

Clearly, $\frac{d^2P}{dx^2} = -1 < 0$ for all x

Hence, P is maximum when $x = y = 4$. The maximum value of P is given by $P = 4 \times 4 = 16$

13 **(d)**

Given curve is $y = 2x^2 - x + 1$ ...(i)

$\Rightarrow \dfrac{dy}{dx} = 4x - 1$

Since, tangent to the curve is parallel to the given line $y = 3x + 9$. Then, slopes will be equal

$\therefore \quad 4x - 1 = 3$

$\Rightarrow \quad x = 1$

From Eq. (i), $y = 2(1)^2 - 1 + 1 = 2$

Hence, required point is (1, 2)

14 **(d)**

Let $P(x_1, y_1)$ be a point on $y^2 = 2x^3$ such that the tangent at P is perpendicular to the line

$$4x - 3y + 2 = 0$$

$\therefore \left(\frac{dy}{dx}\right)_{(x_1, y_1)} \times \left(\frac{-4}{-3}\right) = -1 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{3}{4}$ ...(i)

Now,

$y^2 = 3x^3$

$\Rightarrow 2y\frac{dy}{dx} = 6x^2 \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{3x_1^2}{y_1}$ ...(ii)

From (i) and (ii), we get

$\frac{3x_1^2}{y_1} = \frac{3}{4} \Rightarrow y_1 = 4x_1^2$ ...(iii)

Since (x_1, y_1) lies on $y^2 = 2x^3$

$\therefore y_1^2 = 2x_1^3$ ...(iv)

Solving (iii) and (iv), we get

$(4x_1)^2 = 2x_1^3 \Rightarrow x_1 = 0, x_1 = 1/8$

Putting the values of x_1 in (iv), we get

$y_1 = 0, y_1 = \pm\frac{1}{16}$

Hence, the required points are (0, 0), (1/8, 1/16), (1/8, $-1/16$)

15 **(d)**

We have,

$x = e^t \cos t$ and $y = e^t \sin t$

$\Rightarrow \frac{dx}{dt} = e^t(\cos t - \sin t)$ and $\frac{dy}{dt} = e^t(\sin t + \cos t)$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t + \cos t}{\cos t - \sin t} \Rightarrow \left(\frac{dy}{dx}\right)_{t=\pi/4} = \infty$

So, tangent at $t = \frac{\pi}{4}$ subtends a right angle with x-axis

16 **(b)**

We have,

$y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 4a\left(1 + \cos\frac{x}{a}\right)$

For points at which the tangents are parallel to x-axis, we must have

$\frac{dy}{dx} = 0$

$\Rightarrow 4a\left(1 + \cos\frac{x}{a}\right) = 0 \Rightarrow \cos\frac{x}{a} = -1 \Rightarrow \frac{x}{a} = (2n + 1)\pi$

For these values of x, we have $\sin\frac{x}{a} = 0$

Putting $\sin\frac{x}{a} = 0$ in (i), we get $y^2 = 4ax$

Therefore, all these points lie on the parabola $y^2 = 4ax$

17 **(b)**

Given $y = x^{5/2}$

$\therefore \frac{dy}{dx} = \frac{5}{2}x^{3/2}, \frac{d^2y}{dx^2} = \frac{15}{4}x^{1/2}$

At $x = 0, \frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0$

and $\frac{d^3y}{dx^3}$ is not defined

When $x = 0, y = 0$

$\therefore$ (0, 0) is a point of inflexion

18 **(a)**

Let $f(x) = 2x + 3y$ and $xy = 6$

$\Rightarrow f(x) = 2x + \frac{18}{x}$

On differentiating w.r.t. x, we get

$f'(x) = 2 - \frac{18}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow 0 = 2 - \frac{18}{x^2} \Rightarrow x = \pm 3$

and $f''(x) = \frac{36}{x^3}$

$\Rightarrow f''(3) = \frac{36}{3^3} > 0$

$\therefore$ At $x = 3, f(x)$ is minimum

The minimum value of $f(x)$ is

$f(3) = 2(3) + 3(2) = 12$

19 **(b)**

Given, $f(x) = x^2 - 2x + 4$

$f'(x) = 2x - 2$

By applying Mean value theorem

$f'(c) = 2c - 2 = 0$

$\Rightarrow c = 1$

20 **(b)**

By the algebraic meaning of Rolle's theorem between any two roots of a polynomial there is always a root of its derivative

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	D	A	C	A	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	D	D	D	B	B	A	B	B