DPP

DAILY PRACTICE PRODUCTS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 2

Topic :-Applications of Derivatives

1 **(c)**

From mean value theorem $f'(c)=\frac{f(b)-f(a)}{b-a}$

Given, $a=0 \Rightarrow f(a)=0$

and $b=\frac{1}{2} \quad \Rightarrow f(b)=\frac{3}{8}$

Now, $f'(x)=(x-1)(x-2)+x(x-2)+x(x-1)$

$\therefore\ f'(c)=(c-1)(c-2)+c(c-2)+c(c-1)$

$=c^2-3c+2+c^2-2c+c^2-c$

$\Rightarrow f'(c)=3c^2-6c+2$

By definition of mean value theorem

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow\ 3c^2-6c+2=\frac{\left(\frac{3}{8}\right)-0}{\left(\frac{1}{2}\right)-0}=\frac{3}{4}$$

$$\Rightarrow 3c^2-6c+\frac{5}{4}=0$$

This is a quadratic equation in c

$$\therefore\ c=\frac{6\pm\sqrt{36-15}}{2\times 3}=\frac{6\pm\sqrt{21}}{6}=1\pm\frac{\sqrt{21}}{6}$$

Since, $'c'$ lies between $\left[0,\frac{1}{2}\right]$

$$\therefore\ c=1-\frac{\sqrt{21}}{6}\quad\left(\text{neglecting } c=1+\frac{\sqrt{21}}{6}\right)$$

2 **(a)**

$$\because\ f(x)=\frac{1}{4x^2+2x+1}$$

On differentiating w.r.t. x, we get

$f'(x) = \frac{-(8x+2)}{(4x^2+2x+1)^2}$ ...(i)

For maxima or minima put $f'(x) = 0$

$\Rightarrow\ 8x + 2 = 0 \ \Rightarrow\ x = -\frac{1}{4}$

Again differentiating w.r.t. x of Eq. (i), we get

$$f''(x) = -\frac{\left[\begin{matrix}(4x^2+2x+1)^2(8) - (8x-2)2\times \\ (4x^2+2x+1)(8x+2)\end{matrix}\right]}{(4x^2+2x+1)^4}$$

At $x = -\frac{1}{4}, f''\left(-\frac{1}{4}\right) = -\text{ve}$

$f(x)$ is maximum at $x = -\frac{1}{4}$

$\therefore$ maximum value of $f(x)$

$$f\left(-\frac{1}{4}\right)_{\max} = \frac{1}{4\times\frac{1}{16} - 2\times\frac{1}{4} + 1}$$

$$= \frac{1}{\frac{1}{4} - \frac{2}{4} + 1}$$

$$= \frac{4}{1-2+4} = \frac{4}{3}$$

4 **(b)**

Let $f(x) = 2x + 3y$

$\therefore\ f(x) = 2x + \frac{18}{x}$ $[\because xy = 6, \text{given}]$

$\Rightarrow\ f'(x) = 2 - \frac{18}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\Rightarrow\ 0 = 2 - \frac{18}{x^2}\ \Rightarrow\ x = \pm 3$

And $f''(x) = \frac{36}{x^3}\ \Rightarrow f''(3) = \frac{36}{3^3} > 0$

$\therefore$ At $x = 3$, $f(x)$ is minimum.

The minimum value is $f(3) = 12$

5 **(c)**

On differentiating given curve w. r. t. x, we get

$4\,y^3\frac{dy}{dx} = 3ax^2$

$\Rightarrow\left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$

$\therefore$ Equation of normal at point (a, a) is

$y - a = -\frac{4}{3}\ (x - a)$

$\Rightarrow\ 4x + 3y = 7a$

7 **(b)**

Let $f(x) = x^3 - px + q$

Then $f'(x) = 3x^2 - p$

Put $f'(x) = 0 \Rightarrow x = \sqrt{\frac{p}{3}}, -\sqrt{\frac{p}{3}}$

Now, $f''(x) = 6x$

$\therefore$ At $x = \sqrt{\frac{p}{3}}, f''(x) = 6\sqrt{\frac{p}{3}} > 0,$ minima

And at $x = -\sqrt{\frac{p}{3}}, f''(x) < 0$, maxima

8 **(a)**

For $x = p, y = ap^2bp + c$

And for $x = q, y = aq^2 + bq + c$

Slope $= \frac{aq^2 + bq + c - ap^2 - bp - c}{q - p}$

$= a(q + p) + b$

$\frac{dy}{dx} = 2ax + b = a(q + p) + b$

(according to the equation)

$\therefore x = \frac{q + p}{2}$

9 **(c)**

Clearly, $f(x)$ is continuous and differentiable on the intervals [0, 3] and (0, 3) respectively for all $n \in N$

Also, $f(0) = f(3) = 0$

It is given that Rolle's theorem for the function $f(x)$ defined on [0, 3] is applicable with $c = \frac{3}{4}$

$\therefore f'(c) = 0$

$\Rightarrow 2(c - 3)^n + 2nc(c - 3)^{n-1} = 0$

$[\because f(x) = 2x(x - 3)^n \therefore f'(x) = 2(x - 3)^n + 2nx(x - 3)^{n-1}]$

$\Rightarrow 2(c - 3)^{n-1}(c - 3 + nc) = 0$

$\Rightarrow \frac{3}{4} - 3 + \frac{3}{4}n = 0 \Rightarrow n = 3 \quad \left[\because c = \frac{3}{4}\right]$

10 **(a)**

Given,

$y - x = 1$

$\Rightarrow y = x + 1$

$\frac{dy}{dx} = 1$

And $y^2 = x$

$\Rightarrow 2y\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

$\therefore \quad 1 = \dfrac{1}{2y}$

$\Rightarrow \quad 2y = 1$

$\Rightarrow \quad y = \dfrac{1}{2}$

$\therefore$ point on the curve is $\left(\frac{1}{4}, \frac{1}{2}\right)$

$\therefore$ required shortest distance

$= \left|\dfrac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}}\right| = \dfrac{3}{4\sqrt{2}} = \dfrac{3\sqrt{2}}{8}$

11 **(d)**

Since, $s^3 \propto v \Rightarrow \frac{ds}{dt} = ks^3$.......(i)

$$\Rightarrow \quad \frac{d^2s}{dt^2} = 3ks^2\frac{ds}{dt}$$

$$\Rightarrow \quad \frac{d^2s}{dt} = 3k^2s^5 \qquad [from\ Eq.(i)]$$

Hence, acceleration of particle is proportional to s^5.

12 **(a)**

Given, $s = t^3 + 2t^2 + t$

$\Rightarrow \quad v = \dfrac{ds}{dt} = 3t^2 + 4t + 1$

Speed of the particle after 1 s

$v_{(t=1)} = \left(\dfrac{ds}{dt}\right)_{(t=1)}$

$= 3 \times 1^2 + 4 \times 1 + 1 = 3 + 5 = 8$ cm/s

14 **(b)**

Given, $y = a^x \quad \Rightarrow \quad \frac{dy}{dx} = a^x \log a$

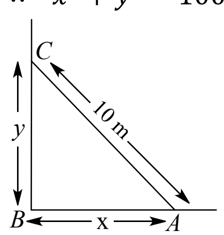
Now, length of subtangent at any point (x_1, y_1)

$= \dfrac{y}{dy/dx} = \dfrac{a^{x_1}}{a^{x_1}\log a} = \dfrac{1}{\log a}$

16 **(c)**

Let $AB = x$m, $BC = y$m and $AC = 10$m

$\therefore \quad x^2 + y^2 = 100 \quad$ ...(i)



On differentiating w.r.t t,m we get

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$
$\Rightarrow\ 2x(3)-2y(4)=0$

$\Rightarrow\ \ x=\frac{4y}{3}$

On putting this value in Eq. (i), we get

$$\frac{16}{9}y^2+y^2=100$$
$\Rightarrow\ \ y^2=\frac{100\times 9}{25}=36\ \ \Rightarrow\ \ y=6\text{m}$

17 **(c)**

Since $f(x)$, which is of degree 3, has relative minimum/maximum at $x=-1$ and $x=\frac{1}{3}$.

Therefore $x=-1, x=\frac{1}{3}$ are roots of $f'(x)=0$. Thus, $x+1$ and $3x-1$ are factors of $f'(x)$

Consequently, we have

$f'(x)=\lambda(x+1)(3\,x-1)=\lambda(3\,x^2+2\,x-1)$

$\Rightarrow f(x)=\lambda\left(x^3+x^2-x\right)+c$

Now, $f(-2)=0$ [Given]

$\Rightarrow c=2\lambda$ ...(i)

We have,

$$\int_{-1}^{1}f(x)dx=\frac{14}{3}$$
$$\Rightarrow\int_{-1}^{1}\left[\lambda\left(x^3+x^2-x\right)+c\right]dx=\frac{14}{3}$$
$\Rightarrow\lambda\int_{-1}^{1}x^2+\int_{-1}^{1}c\,dx=\frac{14}{3}\Rightarrow\frac{2\lambda}{3}+2\,c=\frac{14}{3}\Rightarrow\lambda+3\,c=7$ ...(ii)

Solving (i) and (ii), we get $\lambda=1, c=2$

Hence, $f(x)=x^3+x^2-x+2$

18 **(d)**

$$f(x)=cosx+\frac{1}{2}cos2x-\frac{1}{3}cos3x$$
$\Rightarrow\ \ f'(x)=-\sin x\text{-}\sin 2x\text{+}\sin 3x$

Put $f'(x)=0$

$$\Rightarrow 2\sin\frac{3x}{2}\cos\frac{x}{2}=2\sin\frac{3x}{2}\cos\frac{3x}{2}$$
$$\Rightarrow\ \sin\frac{3x}{2}=0,\cos\frac{3x}{2}=\ \cos\frac{x}{2}$$
$\Rightarrow\ \ x=\frac{2n\pi}{3},\frac{3x}{2}=2n\pi\pm\frac{x}{2}$

$\Rightarrow$ At $x=0,\frac{2\pi}{3},$

At $x=0$

$$f(x)=1+\frac{1}{2}-\frac{1}{3}=\frac{7}{6}$$
At $x=\frac{2\pi}{3},$

$f(x) = -\frac{1}{2} - \frac{1}{4} - \frac{1}{3} = -\frac{13}{12}$

$\therefore$ difference $= \frac{7}{6} + \frac{13}{12} = \frac{27}{12} = \frac{9}{4}$

19 **(a)**

Let $f(x) = e^{x-1} + x - 2$. Then,

$f(1) = e^0 + 1 - 2 = 0$

$\Rightarrow x = 1$ is a real roots of the equation $f(x) = 0$

Let $x = \alpha$ be other real root of $f(x) = 0$ such that $\alpha > 1$

Consider the interval $[1, \alpha]$

Clearly, $f(1) = f(\alpha) = 0$

So, by Rolle's theorem $f'(x) = 0$ has a root in $(1, \alpha)$

But, $f'(x) = e^{x-1} + 1 > 1$ for all x

$\therefore f'(x) \neq 0$ for any $x \in (1, \alpha)$

This is a contradiction

Hence, $f(x) = 0$ has no real root other than 1

20 **(d)**

We have,

$f(x) = \sin^6 x + \cos^6 x$

$\Rightarrow f(x) = (\sin^2 x + \cos^2 x)^3 - 3(\sin^2 x + \cos^2 x)(\sin^2 x \cos^2 x)$

$\Rightarrow f(x) = 1 - 3(\sin^2 x \cos^2 x) = 1 - \frac{3}{4}\sin^2 2x$

$\therefore f(x) \leq 1$, for all x $\left[\because -\frac{3}{4}\sin^2 2x < 0\right]$

and, $f(x) > 1 - \frac{3}{4} = \frac{1}{4}$ for all x $\left[\because -\frac{3}{4}\sin^2 2x \geq -\frac{3}{4}\right]$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10

A.	C	A	C	B	C	A	B	A	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	D	B	C	C	C	D	A	D