DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 9

Topic :-Applications of Derivatives

1 **(c)**

Let r be the base radius and h be the height of the cone. Then, $2r = h$. Let V be the volume of the cone. Then,

$$V = \frac{1}{3}\pi r^2 h = \frac{4\pi r^3}{3}$$

$$\Rightarrow \frac{dV}{dr} = 4\pi r^2$$

$$\therefore \Delta V = \frac{dV}{dr}\Delta r$$

$$\Rightarrow \Delta V = 4\pi r^2 \Delta r$$

$$\Rightarrow \frac{\Delta V}{V} \times 100 = \frac{4\pi r^2 \Delta r}{\frac{4}{3}\pi r^3} \times 100 = 3\left(\frac{\Delta r}{r} \times 100\right) = 3\lambda$$

2 **(b)**

Given, $f'(x) < 0, \forall x \in R$

$$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0, \forall x \in R$$

$$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a, \forall x \in R$$

$$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a, \forall x \in R$$

$$\Rightarrow a \geq 1 \left[\because \sin\left(x + \frac{\pi}{3}\right) \leq 1\right]$$

3 **(d)**

We have,

$$f(x) = \cos\left(\frac{\pi}{x}\right) \Rightarrow f'(x) = \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right)$$

For $f(x)$ to be increasing, we must have

$$f'(x) > 0$$

$$\Rightarrow \frac{\pi}{x^2}\sin\left(\frac{\pi}{x}\right) > 0$$

$$\Rightarrow \sin\left(\frac{\pi}{x}\right) > 0$$

$$\Rightarrow 2n\pi < \frac{\pi}{x} < (2n+1)\pi$$

$\Rightarrow \dfrac{1}{2n} > x > \dfrac{1}{2n+1} \Rightarrow x \in \left(\dfrac{1}{2n+1}, \dfrac{1}{2n}\right)$

4 **(d)**

Given curves are $\frac{x^2}{a^2} + \frac{y^2}{12} = 1$

$\Rightarrow \quad \dfrac{2x}{a^2} + \dfrac{2y}{12} \cdot \dfrac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{12x}{a^2 y} = m_1 \qquad$ (say)

And $y^3 = 8x \Rightarrow 3y^2 \frac{dy}{dx} = 8$

$\Rightarrow \quad \frac{dy}{dx} = \frac{8}{3y^2} = m_2 \qquad$ (say)

For $\quad \theta = \frac{\pi}{2}, 1 + m_1 m_2 = 0$

$\Rightarrow \; 1 + \left(\dfrac{-12x}{a^2 y}\right)\left(\dfrac{8}{3y^2}\right) = 0$

$\Rightarrow \; 3a^2(8x) - 96x = 0$

$\Rightarrow \; a^2 = 4$

5 **(b)**

Consider the function $\phi(x) = f(x) - 2\, g(x)$ defined on [0, 1]

As $f(x)$ and $g(x)$ are differentiable for $0 \le x \le 1$. Therefore, $\phi(x)$ is differentiable on (0, 1) and continuous on [0, 1]

We have,

$\phi(0) = f(0) - 2\, g(0) = 2 - 0 = 2$

$\phi(1) = f(1) - 2\, g(1) = 6 - 2\, g(1)$

Now, $\phi'(x) = f'(x) - 2\, g'(x)$

$\Rightarrow \phi'(c) = f'(c) - 2\, g'(c) = 0$ [Given]

Thus, $\phi(x)$ satisfies Rolle's theorem on [0, 1]

$\therefore \phi(0) = \phi(1)$

$\Rightarrow 2 = 6 - 2\, g(1) \Rightarrow g(1) = 2$

6 **(b)**

$\phi'(x) = f'(x) + a$

$\because \; \phi'(0) = 0$

$\Rightarrow \; f'(0) + a = 0$

$\Rightarrow \; a = 0 \quad (\because f'(0) = 0)$

Also, $\phi'(0) > 0 \quad (\because f''(0) > 0)$

$\Rightarrow \phi'(x)$ has relative minimum at $x = 0$ for all b if $a = 0$

7 **(b)**

Given curve is $y = 2x^2 - x + 1$

On differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = 4x - 1$

Since, this is parallel to the given curve $y = 3x + 9$

$\therefore$ These slopes are equal

$\Rightarrow \; 4x - 1 = 3 \; \Rightarrow \; x = 1$

At $x = 1,\ y = 2(1)^2 - 1 + 1 \Rightarrow y = 2$

Thus, the point is (1, 2).

8 **(a)**

Given, $y = -x^3 + 3x^2 + 2x - 27$

$$\Rightarrow \quad \frac{dy}{dx} = -3x^2 + 6x + 2$$

Let slope $z = \frac{dy}{dx} = -3x^2 + 6x + 2$

Then, $\frac{dz}{dx} = -6x + 6$

For maximum or minimum put $\frac{dz}{dx} = 0$

$\Rightarrow \ -6x + 6 = 0$

$\Rightarrow \ x = 1$

Now, $\frac{d^2z}{dx^2} = -6 < 0$, maxima

The maximum value of z at $x = 1$, is given by

$z = -3 + 6 + 2 = 5$

9 **(b)**

For the curve $y^n = a^{n-1}x$

$$ny^{n-1}y.\frac{dy}{dx} = a^{n-1}$$

$\therefore$ Length of subnormal$= y = \frac{dy}{dx}$

$$= y \times \frac{a^{n-1}}{ny^{n-1}} = \frac{a^{n-1}}{ny^{n-2}}$$

For constant subnormal, n should be 2

10 **(d)**

We have,

$f(x) = 2\,x^2 - \log|x|$

$$\Rightarrow f(x) = \begin{cases} 2\,x^2 - \log x, & x > 0 \\ 2\,x^2 - \log(-x), & x < 0 \end{cases}$$

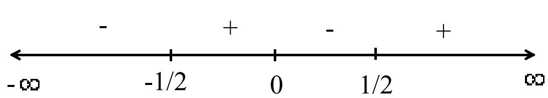
$$\Rightarrow f''(x) = 4\,x - \frac{1}{x} \text{ for all } x \neq 0$$

For $f(x)$ to be increasing, we must have

$f''(x) > 0$

$$\Rightarrow 4\,x - \frac{1}{x} > 0$$

$$\Rightarrow \frac{4\,x^2 - 1}{x} > 0$$



$$\Rightarrow \frac{(2x-1)(2x+1)}{x} > 0$$

$\Rightarrow x(2x-1)(2x+1) > 0$

$\Rightarrow x \in (-1/2, 0) \cup (1/2, \infty)$

11 **(b)**

Let $P(x_1, y_1)$ be the point on the curve $ay^2 = x^3$ where the normal cuts off equal intercepts from the coordinate axes. Therefore,

Slope of the normal at $P = \pm 1$

$$\Rightarrow -\frac{1}{\left(\frac{dy}{dx}\right)_P} = \pm 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_P = \pm 1$$

$$\Rightarrow \frac{3x_1^2}{2ay_1} = \pm 1 \qquad \left[\because ay^2 = x^3 \Rightarrow 2ay\frac{dy}{dx} = 3x^2\right]$$

$$\Rightarrow 9x_1^4 = 4a^2y_1^2$$

$$\Rightarrow 9x_1^4 = 4ax_1^3 \quad [\because (x_1, y_1) \text{ lies on } ay^2 = x^3 = 0 \ \therefore ay_1^2 = x_1^3]$$

$$\Rightarrow x_1 = 0, x_1 = \frac{4a}{9}$$

At $(x_1 = 0, y_1 = 0)$, the normal is y-axis

So, the required point is (x_1, y_1), where $x_1 = \frac{4a}{9}$

12 **(d)**

$\because \ f(x) = \sin x - \cos x$

On differentiating w.r.t. x, we get

$f'(x) = \cos x + \sin x$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right)$$

$$= \sqrt{2}\left[\cos\left(x - \frac{\pi}{4}\right)\right]$$

For decreasing, $f'(x) < 0$

$\frac{\pi}{2} < \left(x - \frac{\pi}{4}\right) < \frac{3\pi}{2}$ (within $0 \le x \le 2\pi$)

$$\Rightarrow \frac{\pi}{2} + \frac{\pi}{4} < \left(x - \frac{\pi}{4} + \frac{\pi}{4}\right) < \frac{3\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

13 **(d)**

Since, $f(x) = \frac{x}{2} + \frac{2}{x}$

$$\therefore \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

For maxima or minima, put $f'(x) = 0$

$$\therefore \quad \frac{1}{2} - \frac{2}{x^2} = 0 \quad \Rightarrow \quad x = \pm 2$$

Now, $f''(x) = \frac{4}{x^3}$

$\Rightarrow \quad f''(2) = \frac{4}{8} = \frac{1}{2} > 0$, minima

And $\quad f''(-2) = -\frac{4}{8} = -\frac{1}{2} < 0$, maxima

Hence, $f(x)$ has local minimum at $x = 2$

14 **(b)**

We have,

$f(x) = \tan^{-1} x - \frac{1}{2} \log_e x$

$\Rightarrow f'(x) = \frac{1}{1+x^2} - \frac{1}{2x}$

$\Rightarrow f'(x) = \frac{2x - 1 - x^2}{2x(1+x^2)}$

$\therefore f'(x) = 0 \Rightarrow 2x - 1 - x^2 = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$

Now,

$f(1) = \tan^{-1} 1 = \frac{\pi}{4}, f\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} - \frac{1}{2} \log_e \left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} + \frac{1}{4} \log_e 3$

and, $f(\sqrt{2}) = \frac{\pi}{3} - \frac{1}{4} \log_e 3$

Hence, the least value of $f(x)$ is $\frac{\pi}{3} - \frac{1}{4} \log_e 3$

15 **(b)**

The given equation of curve is $xy = 1$

$\Rightarrow \quad x\frac{dy}{dx} = -y \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$

Let $\left(t, \frac{1}{t}\right)$ be any point on the curve at which normal to the curve can be drawn

$\therefore \quad \left(\frac{dy}{dx}\right)_{\left(t, \frac{1}{t}\right)} = -\frac{1}{t^2}$

So, slope of normal $= t^2$

Therefore, the given line $ax + bx + c = 0$ will be normal to the curve, if

$t^2 = \frac{-b}{a}$

Since, $\quad t^2 > 0$

$\therefore \quad$ Either $\quad b > 0, \quad a < 0$

or $\quad a > 0, \quad b < 0$

16 **(b)**

Given, $\frac{dy}{dt} \propto y, where$ y is the position of village

$$\Rightarrow \frac{1}{y} dy = k\, dt$$

$\Rightarrow \quad \log y = \log c + kt \qquad$ [on integrating]

$$\Rightarrow \quad \log \frac{y}{c} = kt \Rightarrow y = ce^{kt}$$

17 **(a)**

Given , $x^2 - 2xy + y^2 + 2x + y - 6 = 0$

On differentiating w.r.t.x, we get

$$2x-2\left(y+x\frac{dy}{dx}\right)+2y\frac{dy}{dx}+2+\frac{dy}{dx}=0$$

At (2,2)

$$4-2\left(2+2\frac{dy}{dx}\right)+4\frac{dy}{dx}+2+\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{dy}{dx}=-2$$

$\therefore$ Equation of tangent at (2,2) is

$(y-2)=-2(x-2)$

$\Rightarrow\ 2x+y=6$

18 **(d)**

Given curves are

$x=a(\theta+sin\theta), y=a(1-cos\theta)$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$$

$$=\frac{a(\sin\theta)}{a(1+cos\theta)}$$

$$=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$=tan\frac{\theta}{2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{2}\right)}=\tan\frac{\pi}{4}=1$$

At $\theta=\frac{\pi}{2}, y=a\left(1-cos\frac{\pi}{2}\right)=a$

$\therefore$ length of normal $=y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$=a\sqrt{1+(1)^2}=\sqrt{2}a$

19 **(a)**

Given,

$f(x)=\sin x(1+cosx)$

$$\Rightarrow \quad f(x)=\sin x+\frac{1}{2}\sin 2x$$

On differentiating w.r.t. x, we get

$f'(x)=\cos x+\cos 2x$

Put $f'(x)=0$

$\cos x+\cos 2x=0$

$$\Rightarrow\ 2\cos\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right)=0$$

$$\Rightarrow\ \cos\frac{x}{2}=0, \quad \cos\frac{3x}{2}=0$$

$\Rightarrow \quad x = \pi, x = \dfrac{\pi}{3}$

Now, $\quad f''(x) = -\sin x - 2\sin 2x$

At $x = \frac{\pi}{3}, f''(x) = -\sin\frac{\pi}{3} - 2\sin\frac{2\pi}{3}$

$= -\frac{\sqrt{3}}{2} - \sqrt{3} < 0$, maxima

$\therefore$ maximum value

$f\left(\dfrac{\pi}{3}\right) = \sin\left(\dfrac{\pi}{3}\right)\left(1 + \cos\dfrac{\pi}{3}\right)$

$= \dfrac{\sqrt{3}}{2}\left(1 + \dfrac{1}{2}\right)$

$= \dfrac{3\sqrt{3}}{4} = \dfrac{3^{3/2}}{4}$

20 **(c)**

Let $y = 2x^2 + x - 1$

$y = 4x + 1$

For maxima or minima, put $y' = 0$

$\Rightarrow \quad x = -\dfrac{1}{4}$

Now, $\quad y'' = 4 = +$ ve

y is minimum at $x = -\frac{1}{4}$

Thus, minimum value $= 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) - 1 = -\frac{9}{8}$

Alternate

Here $a > 0$

$\therefore$ Minimum value $= \frac{4ac - b^2}{4a}$

$= \dfrac{4 \times 2(-1) - 1}{4 \times 2} = -\dfrac{9}{8}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	D	D	B	B	B	A	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	D	B	B	B	A	D	A	C