DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 8

Topic :-Applications of Derivatives

1 **(b)**

We have,

$y^2 = 4ax$

$\Rightarrow 2y\frac{dy}{dt} = 4a\frac{dx}{dt}$

$\Rightarrow y\frac{dy}{dt} = 2a\frac{dx}{dt}$

$\Rightarrow y\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^2 = 2a\frac{d^2x}{dt^2}$

$\Rightarrow y \times 0 + (\text{Constant})^2 = 2a\frac{d^2x}{dt^2}\ \left[\because \frac{dy}{dt} = \text{Constant}\right]$

$\Rightarrow \frac{d^2x}{dt^2} = \text{Constant}$

$\Rightarrow$ Projection $(x,0)$ of any point (x,y) on X-axis moves with constant acceleration

2 **(b)**

We have,

$\phi(x) = \int_1^x e^{-t^2/2}(1-t^2)dt \Rightarrow \phi'(x) = e^{-x^2/2}(1-x^2)$

Now,

$\phi'(x) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$

Hence, $x = \pm 1$ are points of extremum of $\phi(x)$

3 **(b)**

Given, $x = 80t - 16t^2$

$$\Rightarrow \quad \frac{dx}{dt} = 80 - 32t$$

At maximum height $\frac{dx}{dt} = 0$

$$\therefore \quad t = 25s$$

4 **(a)**

Let $f(x) = ax^2 + bx + 4$

On differentiating w. r. t., we get

$f'(x) = 2ax + b$

For minimum, put $f'(x) = 0 \Rightarrow x = -\frac{b}{2a}$

Since, it is given that at $x = 1$ minimum value is -1

$\therefore \quad 1 = -\frac{b}{2a} \Rightarrow 2a + b = 0$ ...(i)

And $f(1) = a + b + 4 = -1$

$\Rightarrow \quad a + b + 5 = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get $a = 5, b = -10$

5 **(a)**

Given, $g(x)1 + x) - \frac{2x}{2+x}$

$$\therefore f'(x) = \frac{1}{1+x} - \frac{(2+x)2 - 2x}{(2+x)^2} = \frac{x^2}{(1+x)(x+2)^2}$$

Clearly $f'(x) > 0$ for all $x > 0$.

6 **(c)**

Let m be the slope of the curve $y = f(x)$. Then,

$$m = \frac{dy}{dx}$$

$$\Rightarrow m = e^x(\sin x + \cos x)$$

$$\Rightarrow \frac{dm}{dx} = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$$

$$\Rightarrow \frac{dm}{dx} = 2e^x \cos x$$

$$\Rightarrow \frac{d^2m}{dx^2} = 2e^x(\cos x - \sin x)$$

For maximum/minimum value of m, we must have

$\frac{dm}{dx} = 0 \Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

Clearly, $\frac{d^2m}{dx^2} < 0$ for $x = \frac{\pi}{2}$

Hence, m is maximum when $x = \frac{\pi}{2}$

7 **(b)**

We have,

$y = \sqrt{9 - x^2}$ ...(i)

Clearly, y is positive and defined for $x \in [-3, 3]$

For the points whose ordinates and abscissae are same i.e. $y = x$, we have

$$x = \sqrt{9 - x^2} \Rightarrow 2x^2 = 9 \Rightarrow x = \pm\frac{3}{\sqrt{2}}$$

$$\therefore y = \pm\frac{3}{\sqrt{2}} \quad [\because y = x]$$

But, $y > 0$. Therefore, $xy = y = \frac{3}{\sqrt{2}}$

So, the point is $P\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

Now,

$y=\sqrt{9-x^2}\Rightarrow\frac{dy}{dx}=\frac{-x}{\sqrt{9-x^2}}\Rightarrow\left(\frac{dy}{dx}\right)_p=-1$

8 **(a)**

We know that $\cos x$ is decreasing on $(0,\pi/2)$ and

$\sin x<x$ for $0<x<\frac{\pi}{2}$

$\therefore\cos(\sin x)>\cos x$ for $0<x<\frac{\pi}{2}$

Also,

$0<\cos x<1<\frac{\pi}{2}$ for $0<x<\frac{\pi}{2}$

and, $\sin x<x$ for $0<x<\frac{\pi}{2}$

$\therefore\sin(\cos x)<\cos x$ for $0<x<\frac{\pi}{2}$

Hence, $\cos(\sin x)>\cos x$ and $\sin(\cos x)<\cos x$ for

$0<x<\frac{\pi}{2}$

9 **(d)**

Given, $f(b)-f(a)=(b-a)f'(c)$

$\Rightarrow f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{3-2}{9-4}$

$\Rightarrow f'(c)=\frac{1}{5}$

$\Rightarrow\frac{1}{2}c^{-1/2}=\frac{1}{5}$

$\Rightarrow c=\left(\frac{5}{2}\right)^2=6.25$

10 **(a)**

We have,

$f(x)=(2a-3)(x+2\sin 3)+(a-1)(\sin^4 x+\cos^4 x)+\log 2$

$\Rightarrow f'(x)=2a-3+4(a-1)\sin x\cos x(\sin^2 x-\cos^2 x)$

$\Rightarrow f'(x)=2a-3-(a-1)\sin 4x$

If $f(x)$ does not have critical points, then

$f'(x)=0$ must not have any solution in R

$\Rightarrow(2a-3)-(a-1)\sin 4x=0$ must have no solution in R

$\Rightarrow\sin 4x=\frac{2a-3}{a-1}$ must have no solution in R

$\Rightarrow\left|\frac{2a-3}{a-1}\right|>1$

$\Rightarrow\frac{2a-3}{a-1}<-1$ or, $\frac{2a-3}{a-1}>1$

$\Rightarrow\frac{3a-4}{a-1}<0$ or, $\frac{a-2}{a-1}>0$

$\Rightarrow a\in(1,4/3)$ or, $a\in(-\infty,1)\cup(2,\infty)$

$\Rightarrow a\in(-\infty,1)\cup(1,4/3)\cup(2,\infty)$

For $a = 1$, we have

$f'(x) = -1 \neq 0$

$\Rightarrow f(x)$ has no critical point for $a = 1$

Hence, $a \in (-\infty, 4/3) \cup (2, \infty)$

12 **(d)**

Given, curve is $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,1)} = 2 = m_1$ (say)

And $x = y^2 \Rightarrow 1 = 2y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,1)} = \frac{1}{2} = m_2$ (say)

$\therefore$ Angle of intersection at the point (1, 1) is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}} = \frac{3}{4}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

13 **(c)**

Let volume of sphere $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2.(2) \quad \left[\because \frac{dr}{dt} = 2\right]$$

$$\therefore \frac{dV}{dt} = 8\pi(5)^2 = 200\pi\ cm^3/\min \ [\because r = 5cm]$$

14 **(c)**

Let area of circle, $A = \pi r^2$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi.20.2$$

$$\Rightarrow \frac{dA}{dt} = 80\pi\ cm^2/s$$

15 **(d)**

Given, $y = x^3 - 3x^2 - 9x + 5$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6x - 9$

We know that, this equation gives the slope of the tangent to the curve. The tangent is parallel to x-axis.

$\therefore \frac{dy}{dx} = 0 \quad \Rightarrow \quad 3x^2 - 6x - 9 = 0$

$$\Rightarrow \quad x = -1, 3$$

16 **(b)**

$\because f(x) = x^3 - 6x^2 + 9x + 3$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 - 12x + 9$

$\Rightarrow f'(x) = 3(x^2 - 4x + 3)$

For decreasing, $f'(x) < 0$

$\Rightarrow\ (x-3)(x-1) < 0,$

$\therefore\ \ x \in (1, 3)$

17 **(a)**

We have,

$y = ax^3 + bx^2 + cx$ ...(i)

$\Rightarrow \frac{dy}{dx} = 3ax^2 + 2bx + c$ ...(ii)

It is given that

$\left(\frac{dy}{dx}\right)_{(0,\,0)} = \tan 45° \Rightarrow c = 1$

Also,

$\left(\frac{dy}{dx}\right)_{(1,\,0)} = 0 \Rightarrow 3a + 2b + c = 0 \Rightarrow 3a + 2b + 1 = 0\ \ [\because c = 1]$

Clearly, $a = 1$ and $b = -2$ satisfy this equation

Hence, $a = 1, b = -2$ and $c = 1$

18 **(c)**

Let $f(x) = 1 + x\log_e(x + \sqrt{x^2+1}) - \sqrt{1+x^2}$

Clearly, $f(x)$ is defined for all $x \in R$

Now,

$f'(x) = \log_e\left(x + \sqrt{x^2+1}\right) + \frac{x}{\sqrt{x^2+1}} - \frac{x}{\sqrt{x^2+1}}$

$\Rightarrow f'(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

$\Rightarrow f'(x) > 0$ for all $x \in R$ $[\because x + \sqrt{x^2+1} \geq 1$ for all $x \in R]$

$\Rightarrow f(x)$ is increasing on R

$\Rightarrow f(x) \geq f(0)$ for all $x \geq 0$

$\Rightarrow 1 + x\log_e\left(x + \sqrt{x^2+1}\right) - \sqrt{1+x^2} \geq 0$ for all $x \geq 0$

$\Rightarrow 1 + x\log_e\left(x + \sqrt{x^2+1}\right) \geq \sqrt{1+x^2}$ for all $x \geq 0$

19 **(c)**

Given, $f(x) = x^{-x}$

$\Rightarrow\ \ \log f(x) = -x \log x$

On differentiating w.r.t. x, we get

$\frac{1}{f(x)}.f'(x) = -\log x - 1$

$\Rightarrow\ f'(x) = -f(x)(1 + \log x)$

Put $f'(x) =)0$

$\Rightarrow\ \log x = -1\ \ \Rightarrow\ \ x = e^{-1}$

$\therefore\ \ f''(x) = -f'(x)(1 + \log x) - f(x)\frac{1}{x}$

$= f(x)(1 + \log x)^2 - \frac{f(x)}{x}$

At $x = \frac{1}{e}$,

$f''(x) = -ef\left(\frac{1}{e}\right) < 0$, maxima

Hence, at $x = \frac{1}{e}$, $f(x)$ is maximum.

20 **(b)**

$f(x) = x(x-1)^2$

$f'(x) = 2x(x-1) + (x-1)^2$

$= (x-1)(2x + x - 1) = (x-1)(3x-1)$

$\therefore \quad f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$

$\Rightarrow \quad (c-1)(3c-1) = \dfrac{2-0}{2} = 1$

$\Rightarrow \quad 3c^2 - 4c = 0$

$\Rightarrow \quad c(3c-4) = 0$

$\Rightarrow \quad c = 0 \quad or \quad c = \frac{4}{3}$

$\therefore$ The value of c in (0, 2) is $\frac{4}{3}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	A	C	B	A	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	C	D	B	A	C	C	B