DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject :MATHS
DPP No. : 7

Topic :-Applications of Derivatives

1 **(a)**

Given that, $s=\sqrt{t}$

$\therefore \dfrac{ds}{dt}=\dfrac{1}{2\sqrt{t}}$

$\Rightarrow v=\dfrac{1}{2\sqrt{t}} \Rightarrow \dfrac{dv}{dt}=-\dfrac{1}{2.2t^{3/2}}$

$\Rightarrow a=-\dfrac{2}{(2\sqrt{t})^3}$

$\Rightarrow a=-2v^3$

$\Rightarrow a \propto v^3$

2 **(d)**

Let $PQ=a$ and $PR=b$, then $\Delta=\frac{1}{2}ab\sin\theta$

$\because -1\leq \sin\theta \leq 1$

$\therefore$ Area is maximum when $\sin\theta=1 \Rightarrow \theta=\frac{\pi}{2}$

3 **(a)**

$f'(x)=-a\sin x+b\sec^2 x+1$

Now, $f'(0)=0$ and $f'\left(\frac{\pi}{6}\right)=0$

$\Rightarrow b+1=0$ and $-\frac{a}{2}+\frac{4b}{3}+1=0$

$\Rightarrow b=-1, a=-\dfrac{2}{3}$

4 **(c)**

Given curve is

$y=e^{2x}+x^2$

At $x=0, y=1$

$\therefore$ Any point on the curve is

$\dfrac{dy}{dx}=2e^{2x}+2x$

Slope of normal at $(0,1)=-\frac{1}{2+0}=-\frac{1}{2}$

$\therefore$ Equation of normal is

$y-1=-\frac{1}{2}(x-0)$
$\Rightarrow 2y-2=-x$
$\Rightarrow\ x+2y-2=0$
Required distance=$\left|\frac{0+0-2}{\sqrt{1+4}}\right|$
$=\frac{2}{\sqrt{5}}$

5 **(d)**
We have, $f(x)=x\,e^{1-x}$
$\Rightarrow f'(x)=e^{1-x}(1-x)<0$ for all $x\in(1,\infty)$
So, $f(x)$ strictly decreases in $(1,\infty)$

6 **(a)**
Let $x_1,x_2\in R$ such that $x_1<x_2$. Then,
$x_1<x_2$
$f(x_1)>f(x_2)$ [$\because f$ is a decreasing function]
$\Rightarrow g(f(x_1))<g(f(x_2))$ [$\because g$ is a decreasing function]
$\Rightarrow gof(x_1)<gof(x_2)$

7 **(d)**
$\because\ y^2=4ax$
$\therefore\ \frac{dy}{dx}=\frac{2a}{y}$
Length of subnormal $=y\frac{dy}{dx}=y\frac{2a}{y}=2a$

8 **(c)**
Given, $x=at^2$ and $y=2at$
$\therefore\ \frac{dx}{dt}=2at$ and $\frac{dy}{dt}=2a$
$\therefore$ Slope of tangent, $\left(\frac{dy}{dx}\right)=\frac{2a}{2at}=\frac{1}{t}$
$\Rightarrow\ \frac{1}{t}=\infty,\ \Rightarrow\ t=0$ [given]
$\therefore$ Point of contact is $(0,0)$

9 **(b)**
We have,
$ay^2=x^3$
$\Rightarrow 2\,ay\,\frac{dy}{dx}=3\,x^2$
$\Rightarrow \frac{dy}{dx}=\frac{3\,x^2}{2ay}$
Let (x_1,y_1) be a point on $ay^2=x^3$. Then,
$ay_1^2=x_1^3$ ...(i)
The equation of the normal at (x_1,y_1) is
$y-y_1=-\frac{2ay_1}{3x_1^2}(x-x_1)$
This meets the coordinate axes at

$A\left(x_1+\frac{3x_1^2}{2a},0\right)$ and $B\left(0,y_1+\frac{2ay_1}{3x_1}\right)$

Since the normal cuts off equal intercepts with the coordinate axes

$\therefore x_1+\frac{3\,x_1^2}{2\,a}=y_1+\frac{2\,ay_1}{3\,x_1}$

$\Rightarrow x_1\frac{(2a+3x_1)}{2a}=y_1\frac{(3x_1+2a)}{3x_1}$

$\Rightarrow 3x_1^2=2ay_1$

$\Rightarrow 9x_1^4=4a^2y_1^2$ ...(ii)

From (i) and (ii), we get

$9x_1^4=4a^2\left(\frac{x_1^3}{a}\right)\Rightarrow x_1=\frac{4a}{9}$

10 **(d)**

Given, $y=2x^2-6x-4\ \Rightarrow\ \frac{dy}{dx}=4x-6$

Since, $\frac{dy}{dx}=0\quad\Rightarrow\quad\frac{dy}{dx}=4x-6=0\quad\Rightarrow\quad x=\frac{3}{2}$

$\Rightarrow\quad y=2.\frac{9}{4}-6.\frac{3}{2}-4=-\frac{17}{2}$

$\therefore$ Required point is $\left(\frac{3}{2},-\frac{17}{2}\right)$

11 **(d)**

$\because f(x)=x^3-6x^2+ax+b$

On differentiating w.r.t. x, we get

$f'(x)=3x^2-12x+a$

By the definition of Rolle's theorem

$f'(c)=0\ \Rightarrow\ f'\left(2+\frac{1}{\sqrt{3}}\right)=0$

$\Rightarrow\ 3\left(2+\frac{1}{\sqrt{3}}\right)^2=-12\left(2+\frac{1}{\sqrt{3}}\right)+a=0$

$\Rightarrow\ 3\left(4+\frac{1}{3}+\frac{4}{\sqrt{3}}\right)-12\left(2+\frac{1}{\sqrt{3}}\right)+a=0$

$\Rightarrow\ 12+1+4\sqrt{3}-24-4\sqrt{3}+a=0$

$\Rightarrow\ a=11$

12 **(b)**

For the point $(3,\log 2)$, we take $y=g(x)=\log(x-1)$

$\frac{dy}{dx}=\frac{1}{(x-1)}\quad\Rightarrow\left(\frac{dy}{dx}\right)_{(3,\log 2)}=\frac{1}{2}$

$\therefore$ Equation of normal is

$y-\log 2=-2(x-3)$

$\Rightarrow\ y+2x=6+\log 2$

14 **(c)**

Let T be the processing time. The number of batches is $\frac{N}{x}$ and the processing time of one batch is $\alpha + \beta\, x^2$ seconds

$\therefore T = \frac{N}{x}(\alpha + \beta\, x^2) = N\left(\frac{\alpha}{x} + \beta\, x\right) \Rightarrow \frac{dT}{dx} = N\left(-\frac{\alpha}{x^2} + \beta\right)$

For fast processing T must be least for which $\frac{dT}{dx} = 0$

$\therefore N\left(-\frac{\alpha}{x^2} + \beta\right) = 0 \Rightarrow x = \sqrt{\frac{\alpha}{\beta}}$

Clearly, $\frac{d^2T}{dx^2} = N\frac{2\alpha}{x^3} > 0$ for $x = \sqrt{\frac{\alpha}{\beta}}$

Hence, T is least when $x = \sqrt{\frac{\alpha}{\beta}}$

15 **(d)**

Let (x, y) be the point on the curve $2x^2 + y^2 - 2x = 0$. Then its distance from $(a, 0)$ is given by

$S = \sqrt{(x - a)^2 + y^2}$

$\Rightarrow S^2 = x^2 - 2\,ax + a^2 + 2\,x - 2\,x^2$ [Using $2\,x^2 + y^2 - 2\,x = 0$]

$\Rightarrow S^2 = -x^2 + 2\,x(1 - a) + a^2$ ...(i)

$\Rightarrow 2\,S\,\frac{dS}{dx} = -2\,x + 2(1 - a)$

For S to be maximum, we must have,

$\frac{dS}{dx} = 0 \Rightarrow -2x + 2(1 - a) = 0 \Rightarrow x = 1 - a$

It can be easily checked that $\frac{d^2S}{dx^2} < 0$ for $x = 1 - a$

Hence, S is maximum for $x = 1 - a$

Putting $x = 1 - a$ in (i), we get $S = \sqrt{1 - 2\,a + 2\,a^2}$

16 **(b)**

Given curve is

$y = x^2 - 3x + 2$

$\frac{dy}{dx} = 2x - 3$

$(2x - 3) \times 1 = -1$

$\Rightarrow \quad 2x - 3 = -1$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

$\therefore \quad y = 0$

$\Rightarrow$Required point is(1,0).

17 **(c)**

The equations of the tangent and normal to $y^2 = 4ax$ at $P(at^2, 2at)$ are

$ty = x + at^2$ ...(i)

and, $y + tx = 2\,at + at^3$ ...(ii)

Liens (i) and (ii) meet the x-axis at $T(-at^2, 0)$ and $G(2\,a + at^2, 0)$ respectively

Since PT is perpendicular to PG. Therefore, TG is the diameter of the circle through P, T, G

Hence, the equation of the circle is

$(x + at^2)(x - 2a - at^2) + (y - 0)(y - 0) = 0$

$\Rightarrow x^2 + y^2 - 2ax - at^2(2a + at^2) = 0$

$\Rightarrow 2x + 2y\dfrac{dy}{dx} - 2a = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{a - x}{y}$

$\Rightarrow m_1 = \left(\frac{dy}{dx}\right)_P = \frac{a - at^2}{2at} = \frac{1 - t^2}{2t}$ ...(i)

$\Rightarrow y^2 = 4ax$

$\Rightarrow \left(\dfrac{dy}{dx}\right) = \dfrac{2a}{y}$

$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_P = \frac{2a}{2at} = \frac{1}{t}$ ...(ii)

Let θ be the angle between the tangents at P to the parabola and the circle. Then,

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{\frac{1}{t} - \frac{1 - t^2}{2t}}{1 + \frac{1 - t^2}{2t^2}} = t \Rightarrow \theta = \tan^{-1} t$$

18 **(a)**

Given, $x = a(\cos\theta + \theta\sin\theta)$

And $y = a(\sin\theta - \theta\cos\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \dfrac{dx}{d\theta} = a\theta\cos\theta$

And $\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta)$

$\Rightarrow \dfrac{dy}{d\theta} = a\theta\sin\theta$

$\therefore \dfrac{dy}{dx} = \tan\theta$

So, equation of normal is

$\Rightarrow y - a\sin\theta + a\theta\cos\theta = -\dfrac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$

$y\sin\theta - a\sin^2\theta + a\theta\cos\theta\sin\theta$

$= -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$

$\Rightarrow x\cos\theta + y\sin\theta = a$

It is always at a constant distance 'a' from origin.

19 **(a)**

$\because f(x) = xe^{x(1-x)}$

On differentiating w.r.t. x, we get

$f'(x) = e^{x(1-x)+x.e^{x(1-x)}}.(1-2x)$

$= e^{x(1-x)}\{1 + x(1-2x)\}$

$= e^{x(1-x)}.(-2x^2 + x + 1)$

It is clear that $e^{x(1-x)} > 0$ for all x

Now, by sign rule for $-2x^2 + x + 1$

$f'(x) > 0$, if $x \in \left[-\frac{1}{2}, 1\right]$

So, $f(x)$ is increasing on $\left[-\frac{1}{2}, 1\right]$

20 **(a)**

$\because \quad f(x) = 2x^3 - 15x^2 + 36x + 4$

On differentiating w.r.t. x, we get

$f'(x) = 6x^2 - 30x + 36 \quad ...(i)$

For maxima or minima $f'(x) = 0$

$\Rightarrow \quad 6x^2 - 30x + 36 = 0$

$\Rightarrow \; x^2 - 5x + 6 = 0$

$\Rightarrow \quad (x-2)(x-3) = 0 \quad \Rightarrow \quad x = 2, 3$

Again differentiating Eq. (i), we get

$f''(x) = 12x - 30$

$\Rightarrow \quad f''(2) = 24 - 30 = -6 < 0$

Therefore, $f(x)$ is maximum at, $x = 2$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	A	C	D	A	D	C	B	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	B	B	C	D	B	C	A	A	A