DPP

DAILY PRACTICE PROBLEMS

Class : XIth | **Solutions** | **Subject :MATHS**

Date : | | **DPP No. : 10**

Topic :-Applications of Derivatives

1 **(a)**

$\because \quad x = t^2$ and $y = 2t$

$\therefore$ At $t = 1,\ x = 1$ and $y = 2$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{2t} = \frac{1}{t}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,2)} = 2$

$\therefore$ Equation of normal is

$y - 2 = -1(x - 1)$

$\Rightarrow \quad x + y - 3 = 0$

3 **(d)**

Let $y = a\sec\theta - b\tan\theta$

$\Rightarrow \frac{dy}{d\theta} = a\,sec\theta tan\theta - bsec^2\theta$

Put $\frac{dy}{d\theta} = 0 \Rightarrow sec\theta(a\,tan\theta - bsec\theta) = 0$

$\Rightarrow sin\theta = \frac{b}{a} \qquad (\because sec\theta \neq 0)$

Now, $\frac{d^2y}{d\theta^2} > 0, at\ sin\theta = \frac{b}{a}$

$\therefore$ minimum value is

$y = a\frac{a}{\sqrt{a^2 - b^2}} - b\frac{b}{\sqrt{a^2 - b^2}}$

$= \sqrt{a^2 - b^2}$

4 **(d)**

We have,

$y = x^n$

$\Rightarrow \log y = n \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{n}{x} \Rightarrow \frac{dy}{dx} = \frac{ny}{x}$

$\therefore \Delta y = \frac{dy}{dx}\Delta x$

$\Rightarrow \Delta y = \frac{ny}{x}\Delta x \Rightarrow \frac{\Delta y}{y} = \left(\frac{\Delta x}{x}\right) \times n \Rightarrow \frac{\Delta y}{y} \div \frac{\Delta x}{x} = n$

5 **(c)**

Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \forall x \in R$

$\therefore f(x)$ is increasing

$\therefore \quad f(x) = 0$ has only one solution

6 **(c)**

Given, $f(x) = x^3 + ax^2 + bx + c, a^2 \leq 3b$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 + 2ax + b$

Put $f'(x) = 0$

$\Rightarrow 3x^2 + 2ax + b = 0$

$\Rightarrow \quad x = \frac{-2a \pm \sqrt{4a^2 - 12b}}{2 \times 3}$

$= \frac{-2a \pm 2\sqrt{a^2 - 3b}}{6}$

Since, $a^2 \leq 3b$,

$\therefore \quad x$ Has an imaginary value.

Hence, no extreme value of x exist.

8 **(c)**

We have,

$xy^n = a^{n+1} \Rightarrow y^n + n\, xy^{n-1}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{nx}$

Let (x_1, y_1) be a point on $x\, y^n = a^{n+1}$. The equation of the tangent at (x_1, y_1) is

$y = y_1 = -\frac{y_1}{nx_1}\ (x - x_1)$

This meets with the coordinate axes at $A((n+1)x_1, 0)$ and $B\left(0, \frac{(n+1)y_1}{n}\right)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}(n+1)x_1 \cdot \left(\frac{n+1}{n}\right)y_1$

$\Rightarrow$ Area of $\Delta OAB = \frac{1}{2}\frac{(n+1)^2}{n}x_1y_1 \quad ...(\text{i})$

Since (x_1, y_1) lies on $xy^n = a^{n+1}$

$\therefore x_1y_1^n = a^{n+1}(x_1, y_1) \Rightarrow x_1 = \frac{a^{n+1}}{y_1^n}$

Putting the value of x_1 in (i), we get

Area of $\Delta OAB = \frac{1}{2}\frac{(n+1)^2}{n}a^{n+1}y_1^{-n+1}$

This will be a constant, if $n = 1$

9 **(a)**

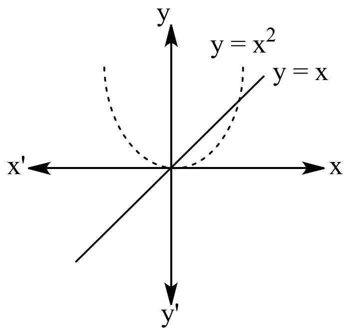
The area of circular plate is $A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi(12)(0.01) = 0.24\pi$ sq cm/s

10 **(a)**

$\therefore g(x) = \min(x, x^2)$



It is clear from the graph that g(x) is an increasing function

11 **(d)**

The point of intersection of given curve is (0, 1). On differentiating given curves, we get

$\frac{dy}{dx} = a^x \log a, \frac{dy}{dx} = b^x \log b$

$\Rightarrow \quad m_1 = a^x \log a, m_2 = b^x \log b$

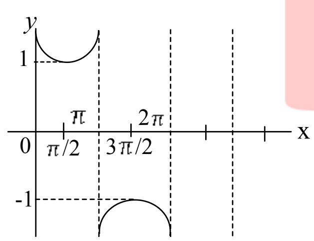
At (0,1) $m_1 = \log a, m_2 = \log b$

$\therefore \quad \tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{\log a - \log b}{1 + \log a \log b}\right|$

12 **(a)**

The graph of cosec x is opposite in interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



13 **(b)**

Given, $f(x) = x^{25}(1-x)^{75}$

$\Rightarrow \quad f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74}$

$= 25x^{24}(1-x)^{74}(1-4x)$

Put, $f'(x) = 0 \quad \Rightarrow \quad x = 0, 1, \frac{1}{4}$

If $x < \frac{1}{4}$, then

$f'(x) = 25x^{24}(1-x)^{74}(1-4x) > 0$

And if $x > \frac{1}{4}$, then

$f'(x) = 25x^{24}(1-x)^{24}(1-4x) < 0$

Thus, $f'(x)$ changes its sign from positive to negative as x passes through 1/4 from left to right.

Hence, $f(x)$ attains its maximum at x=1/4

14 **(d)**

Given, $f(x) = e^x \sin x, \quad x \in [0, \pi]$
At $x = 0, \quad f(0) = 0$
And at $x = \pi, \quad f(\pi) = 0$
Also, it is continuous and differentiable in the given interval.
Hence, it satisfies the Rolle's theorem.
Hence, option (d) is the required answer

15 **(b)**

Given curve is

$$xy = c^2 \Rightarrow y = \frac{c^2}{x^2}$$

Let $f(x) = ax + by = ax + \frac{bc^2}{x^2}$

On differentiating w.r.t. x, we get

$$f'(x) = a - \frac{bc^2}{x^2}$$

For a maximum or minima, put $f'(x) = 0$

$\Rightarrow ax^2 - bc^2 = 0$

$$\Rightarrow x^2 = \frac{bc^2}{a} \Rightarrow x = \pm c\sqrt{\frac{b}{a}}$$

Again on differentiating, we get $f''(x) = \frac{2bc^2}{x^3}$

At6 $x = c\sqrt{\frac{b}{a}}, \ f''(x) > 0$

$\therefore \quad f(x)$ is minimum at $x = c\sqrt{\frac{b}{a}}$

The minimum value at $x = c\sqrt{\frac{b}{a}}$ is

$$\therefore f\left(c\sqrt{\frac{b}{a}}\right) = a.c\sqrt{\frac{b}{a}} + \frac{bc^2}{c}.\sqrt{\frac{a}{b}}$$

$$= \frac{abc + abc}{\sqrt{ab}} = \frac{2abc}{\sqrt{ab}} = 2c\sqrt{ab}$$

16 **(a)**

Given, $x - 2y = 4$

Let $A = xy \Rightarrow A = 2y^2 + 4y$

$$\Rightarrow \frac{dA}{dy} = 4 + 4y$$

For extremum value, $\frac{dA}{dy} = 0$

$\Rightarrow \quad y = -1$

Now, $\frac{d^2A}{dy^2} = 4 > 0$, minima

At $y = -1$,

$x = 4 + 2(-1) = 2$

$\therefore \quad A = xy = 2(-1) = -2$

$\therefore$ Minimum value of xy is -2

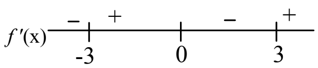
17 **(a)**

Given, $f(x) = (9 - x^2)^2$

$\Rightarrow \; f''(x) = 2(9 - x^2)(-2x)$

Now, put $f''(x) = 0$

$\Rightarrow \; 2(9 - x^2)(-2x) = 0 \;\Rightarrow\; x = 0, \pm 3$



$\therefore \; f''(x)$ is increasing in $(-3, 0) \cup (3, \infty)$

18 **(b)**

Let $f(x) = x + \frac{1}{x} \;\Rightarrow\; f'(x) = 1 - \frac{1}{x^2}$

For maxima and minima, put $f'(x) = 0$

$\Rightarrow \; 1 - \frac{1}{x^2} = 0 \;\Rightarrow\; x = \pm 1$

Now, $f''(x) = \frac{2}{x^3}$

At $x = 1$, $f''(x) = \frac{2}{x^3}$

At $x = 1$, $f''(x) = +$ ve, minima

And at $x = -1$, $f''(x) = -$ ve, maxima

Thus, $f(x)$ attains minimum value at $x = 1$

19 **(a)**

We have,

$12y = x^3$

$\Rightarrow 12\frac{dy}{dt} = 3x^2\frac{dx}{dt}$

$\Rightarrow 12 = 3x^2 \qquad \left[\because \frac{dy}{dt} = \frac{dx}{dt}\right]$

$\Rightarrow x = \pm 2$

$\therefore 12y = x^3 \Rightarrow y = \pm\frac{2}{3}$

Hence, the points are $(2, 2/3)$ and $(-2, -2/3)$

20 **(b)**

Let $R(x_1, y_1)$ be the point on the parabola $y^2 = 2x$ such that tangent at R is parallel to the chord PQ

$\therefore \left(\frac{dy}{dx}\right)_R =$ Slope of PQ

$\Rightarrow \frac{1}{y_1} = \frac{-1-2}{\frac{1}{2} - 2} \qquad \left[\because y^2 = 2x \Rightarrow 2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}\right]$

$\Rightarrow \frac{1}{y_1} = 2 \Rightarrow y_1 = \frac{1}{2}$

Since (x_1, y_1) lies on $y^2 = 2x$

$\therefore y_1^2 = 2x_1 \Rightarrow x_1 = \frac{1}{8} \qquad [\because y = 1/2]$

Hence, the required point is (1/8, 1/2)

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	D	D	C	C	A	C	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	D	B	A	A	B	A	B