DPP
DAILY PRACTICE PROBLEMS

Class : XIth **Subject :MATHS**
Date : **DPP No. : 1**

Solutions

Topic :-Applications of Derivatives

1 **(b)**

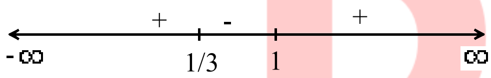
We have,

$f(x) = x(x-1)^2$

$\Rightarrow f''(x) = (x-1)^2 + 2x(x-1)$

$\Rightarrow f''(x) = (x-1)(3x-1)$

The changes in the signs of $f''(x)$ are shown in diagram

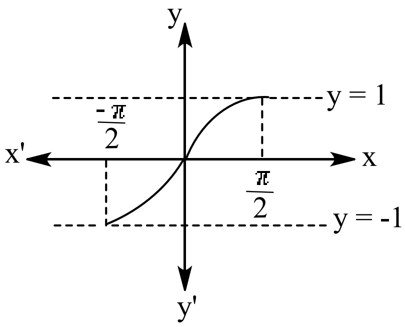


Clearly, $f(x)$ attains a local maximum at $x = \frac{1}{3}$ and a local minimum at $x = 1$

$\therefore$ Maximum value of $f(x) = f\left(\frac{1}{3}\right) = \frac{4}{27}$

2 **(a)**

Since, $2\pi k - \frac{\pi}{2} \leq \sin x \leq 2\pi k + \frac{\pi}{2}$



For $k = 0$

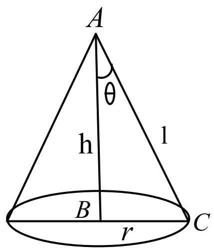
$-\frac{\pi}{2} < \sin x < \frac{\pi}{2}$

Which increase from -1 *to* 1.

Similarly, for other values of k it is increase from -1 *to* 1.

3 **(c)**

Volume of cone, $V = \frac{\pi}{3} r^2 h$



$\Rightarrow \quad V=\frac{\pi}{3}r^2\sqrt{l^2-r^2}$

On differentiating w.r.t. r, we get

$\frac{dV}{dr}=\frac{\pi}{3}\left[2r\sqrt{l^2-r^2}+\frac{r^2}{2\sqrt{l^2-r^2}}(-2r)\right]$

Put $\frac{dV}{dr}=0$

$\Rightarrow \ 2r\sqrt{(l^2-r^2)}-\frac{r^3}{\sqrt{l^2-r^2}}=0$

$\Rightarrow \ r[2(l^2-r^2)-r^2]=0$

$\Rightarrow r=\pm \ l\sqrt{\frac{2}{3}}$

$\therefore$ At $\ r=l\sqrt{\frac{2}{3}}, \frac{d^2V}{dr^2}<0$, maxima

$\therefore \ \ h=\sqrt{l^2-\frac{2}{3}l^2=\frac{l}{\sqrt{3}}}$

In ΔABC, $\tan\theta=\frac{r}{h}=\frac{l\sqrt{\frac{2}{3}}}{\frac{l}{\sqrt{3}}}=\sqrt{2}$

4 **(c)**

Given that $f(x)=\sin x-bx+c$

$\therefore f'(x)=\cos x-b$

For decreasing, $f'(x)<0$,for all $x\in R$.

$\Rightarrow \cos x<b \ for\ all\ x\in R \Rightarrow b>1$.

5 **(c)**

Given, $f(x)=2x^3+3x^2-12x+1$

$\Rightarrow f'(x)=6x^2+6x-12$

For $f(x) to$ be decreasing, $f'(x)<0$

$\Rightarrow 6(x^2+x-2)<0$

$\Rightarrow (x+2)(x-1)<0$

$\Rightarrow x\in(-2,1)$

6 **(a)**

We have,

$f(x)=2\,x+\cot^{-1}x+\log\left(\sqrt{1+x^2}-x\right)$

$\therefore f'(x)=2-\frac{1}{1+x^2}+\frac{1}{\sqrt{1+x^2}-x}\left(\frac{x}{\sqrt{1+x^2}}-1\right)$

$\Rightarrow f'(x)=\frac{1+2\,x^2}{1+x^2}-\frac{1}{\sqrt{1+x^2}}$

$$\Rightarrow f'(x)=\frac{1+2x^2}{1+x^2}-\frac{\sqrt{(1+x^2)}}{1+x^2}$$

$$\Rightarrow f'(x)=\frac{x^2+\sqrt{1+x^2}\{\sqrt{1+x^2}-1\}}{1+x^2}>0 \text{ for all } x$$

Hence, $f(x)$ is an increasing function on $(-\infty,\infty)$ and in particular on $(0,\infty)$

7 **(c)**

We have,

$$f(x)=3\cos^2 x+4\sin^2 x+\cos\frac{x}{2}+\sin\frac{x}{2}$$

$$\Rightarrow f(x)=4-\cos^2 x+\cos\frac{x}{2}+\sin\frac{x}{2}$$

$$\Rightarrow f'(x)=\sin 2x-\tfrac{1}{2}\left(\sin\tfrac{x}{2}-\cos\tfrac{x}{2}\right) \quad ....(i)$$

$$\Rightarrow f'(x)=2\sin x\cos x-\frac{1}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)$$

$$\Rightarrow f'(x)=2\sin x\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)+\frac{1}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)$$

$$\Rightarrow f'(x)=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left\{2\sin x\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\frac{1}{2}\right\}$$

$$\Rightarrow f'(x)=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left\{2\sqrt{2}\sin x\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)+\frac{1}{2}\right\}$$

For local maximum or minimum, we must have

$f'(x)=0$

$$\Rightarrow \cos\frac{x}{2}-\sin\frac{x}{2}=0$$

$$\Rightarrow \cos\frac{x}{2}=\sin\frac{x}{2}\Rightarrow\tan\frac{x}{2}=1\Rightarrow\frac{x}{2}=\frac{\pi}{4}\Rightarrow x=\frac{\pi}{2}$$

Now,

$$f''(x)=2\cos 2x-\tfrac{1}{4}\left(\cos\tfrac{x}{2}+\sin\tfrac{x}{2}\right) \quad \text{[Using (i)]}$$

$$\Rightarrow f''\left(\frac{\pi}{2}\right)=2\cos\pi-\frac{1}{4}\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)=-2-\frac{1}{2\sqrt{2}}<0$$

Thus, $f(x)$ attains a local maximum at $x=\frac{\pi}{2}$

Local maximum value $=f\left(\frac{\pi}{2}\right)=4+\frac{2}{\sqrt{2}}=4+\sqrt{2}$

8 **(c)**

$$\because \quad y=\left(\frac{c^6-a^2x^4}{b^2}\right)^{\frac{1}{4}}$$

Let $f(x)=xy=\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{\frac{1}{4}}$

$$\Rightarrow \quad f'(x)=\frac{1}{4}\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{-3/4}$$

$$\left(\frac{4x^3c^6}{b^2}-\frac{8x^7a^2}{b^2}\right)$$

Put $f'(x)=0$

$$\Rightarrow \quad x=\pm\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$$

$\therefore \quad f\left(\frac{c^{3/2}}{c^{1/4}\sqrt{a}}\right) = \frac{c^3}{\sqrt{2ab}}$

9 **(a)**

Let the radius of the circular wave ring by r cm at any time t. Then, $\frac{dr}{dt} = 30$ cm/sec (given)

Let A be the area of the enclosed ring. Then,

$A = \pi r^2$

$\Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 2\pi \times 50 \times \frac{30}{100} \text{m}^2\text{sec} = 30\pi^2 \text{ m}^2/\text{sec}$

10 **(b)**

We have,

$x = t\cos t$ and $y = t\sin t$

$\therefore \frac{dx}{dt} = \cos t - t\sin t$ and $\frac{dy}{dx} = \sin t + t\cos t$

At the origin, we have

$x = 0, y = 0 \Rightarrow t\cos t = 0$ and $t\cos t = 0 \Rightarrow t = 0$

The slope of the tangent at $t = 0$ is

$$\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)_{t=0} = \left(\frac{\sin t + t\cos t}{\cos t - t\sin t}\right)_{t=0} = 0$$

So, the equation of the tangent at the origin is

$t - 0 = 0(x - 0) \Rightarrow y = 0$

11 **(c)**

Surface area of sphere $S = 4\pi r^2$ and $\frac{dr}{dt} = 2$

$\therefore \quad \frac{dS}{dt} = 4\pi \times 2r\frac{dr}{dt} = 8\pi r \times 2 = 16\pi r$

$\Rightarrow \frac{dS}{dt} \propto r$

12 **(c)**

Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x} = e^{-x\log x}$. Then,

$f'(x) = -\left(\frac{1}{x}\right)^x (\log x + 1) = -x^{-x}(\log x + 1)$

Now,

$f'(x) = 0$

$\Rightarrow -x^{-x}(\log x + 1) = 0$

$\Rightarrow \log x + 1 = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1}$

Clearly, $f''(x) < 0$ at $x = e^{-1}$

Hence, $f(x) = x^{-x}$ is maximum for $x = e^{-1}$. The maximum value is $e^{1/e}$

13 **(c)**

Given, $\quad f(x) = 2x^3 - 21x^2 + 36x - 30$

$\Rightarrow \quad f'(x) = 6x^2 - 42x + 36$

For maxima or minima, put $f'(x) = 0$

$\Rightarrow 6x^2 - 42x + 36 = 0 \quad \Rightarrow \quad x = 6, 1$

And $\quad f''(x) = 12x - 42$

$f''(1) = -30$ and $f''(6) = 30$

Hence, $f(x)$ has maxima at $x = 1$ and minima at $x = 6$

14 **(a)**

Let $l$be the length of an edge and Vbe the volume of cue at any time t.

$\because \quad V = t^3$

$\because \quad \frac{dV}{dt} = 3l^2 \frac{dl}{dt}$

$= 3 \times 5^2 \times 10 cm^3/s$

$= 750 cm^3/s.$

15 **(c)**

We have, $\frac{dy}{dx} = \frac{-\sin\theta}{1-\cos\theta}$

Clearly, $\frac{dy}{dx} = 0$ for $\theta = (2k+1)\pi$

So, the tangent is parallel to x-axis i.e. $y = 0$

16 **(b)**

We have,

$5x^5 - 10x^3 + x + 2y + 6 = 0$ ...(i)

Differentiating with respect to x, we get

$25x^4 - 30x^2 + 1 + 2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}(25x^4 - 30x^2 + 1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,-3)} = -\frac{1}{2}$

The equation of the normal at $(0, -3)$ is

$y + 3 = 2(x - 0) \Rightarrow 2x - y - 3 = 0$

Solving (i) and (ii), we obtain the coordinates of their points of intersection as P $(0, -3), (1, -1)$ and $(-1, -5)$

Hence, the normal at $P(0, -3)$ meets the curve again at $(1, -1)$ and $(-1, -5)$

17 **(b)**

We have,

$\frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\,\theta\cos\theta$

and, $\frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\,\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \tan\theta \Rightarrow -\frac{1}{\frac{dy}{dx}} = -\cot\theta$

Hence, the slope of the normal varies as θ

The equation of the normal at any point is

$y - a(\sin\theta - \theta\cos\theta) = -\cot\theta\ \{x - a(\cos\theta + \theta\sin\theta)\}$

$\Rightarrow x\cos\theta + y\sin\theta = a$

Clearly, it is a line at a constant distance $|a|$ from the origin

18 **(d)**

We have,

$f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$

Clearly, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

So, $f(x)$ is continuous at $x = 2$
Hence, it is continuous on $[-1, 3]$
Thus, option (b) is correct
We find that
$f'(x) = 6x + 12 > 0$ for all $x \in [-1, 2]$
$\Rightarrow f(x)$ is increasing on $[-1, 2]$
Thus, option (a) is correct
Also,
$f'(x) < 0$ for all $x \in (2, 3]$
$\Rightarrow f(x)$ is decreasing on $(2, 3]$
Hence, $f(x)$ is attains the maximum value at $x = 2$
So, option (c) is correct

19 **(b)**

Given, $f(x) = x^3 - 3x^2 + 2x$
$\Rightarrow f'(x) = 3x^2 - 6x + 2$
Now, f(a)=f(0)=0
And $f(b) = f\left(\frac{1}{2}\right)$
$$= \frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right) = \frac{3}{8}$$
By Lagrange's Mean Value Theorem
$$\frac{f(b) - f(a)}{b - a} = f'(c)$$
$$\Rightarrow \frac{\frac{3}{8} - 0}{\frac{1}{2} - 0} = 3c^2 - 6c + 2$$
$\Rightarrow 12c^2 - 24x + 5 = 0$
This is a quadratic equation in c.
$$c = \frac{24 \pm \sqrt{576 - 240}}{24}$$
$$= 1 \pm \frac{\sqrt{21}}{6}$$
But c lies between 0 to $\frac{1}{2}$
$\therefore$ we take, $c = 1 - \frac{\sqrt{21}}{6}$

20 **(a)**

Since, $f(x) = kx - \sin x$ is monotonically increasing for all $x \in R$. Therefore,
$f'(x) > 0$ for all $x \in R$
$\Rightarrow K - \cos x > 0$
$\Rightarrow K > \cos x$
$\Rightarrow K > 1$ [$\therefore$ maximum value of $\cos x$ is 1]

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	C	C	A	C	C	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	C	A	C	B	B	D	B	A