

DPP

DAILY PRACTICE PROBLEMS

CLASS : XITH
DATE :

Solutions

SUBJECT : PHYSICS
DPP NO. : 9

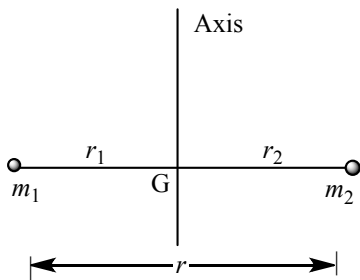
Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1 (d)
Total kinetic energy = $\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = 32.8 J$
 $\Rightarrow \frac{1}{2} \times 10 \times (2)^2 \left(1 + \frac{K^2}{(0.5)^2}\right) = 32.8 \Rightarrow K = 0.4 m$

2 (a)
Kinetic energy of rotating body = $\frac{1}{2}I\omega^2$
 $= \frac{1}{2} \times 3 \times (3)^2 = 13.5 J$
Kinetic energy of translating body = $\frac{1}{2}mv^2$
As both are equal according to problem *i.e.*
 $\frac{1}{2}mv^2 = 13.5 \Rightarrow \frac{1}{2} \times 27 \times v^2 = 13.5 \Rightarrow \therefore v = 1 m/s$

3 (d)
According to conservation of momentum
 $5 \times 10 = (955 + 5)v$
 $v = \frac{50}{100} = \frac{1}{20} \quad \therefore \% \text{ KE lost} = \frac{K_1 - K_2}{K_1} \times 100 = 95.5\%$

4 (d)
Total moment of inertia will be equal to the sum of moment of inertia due to individual masses.



$$I = \sum_{i=1}^n I_i$$

where, $I_1 = m_1 r_1^2, I_2 = m_2 r_2^2$.

Given, $r = r_1 + r_2$

$$m_1 r_1 = m_2 r_2$$

$$\therefore m_1 r_1 = m_2 (r - r_1)$$

$$\Rightarrow m_1 r_1 + m_2 r_1 = m_2 r$$

$$\Rightarrow r_1 (m_1 + m_2) = m_2 r$$

$$\Rightarrow r_1 = \frac{m_2 r}{(m_1 + m_2)}$$

Also, $r_2 = r - r_1$

$$r_2 = r - \frac{m_2 r}{(m_1 + m_2)} = \frac{m_1 r}{m_1 + m_2}$$

$$\therefore I = I_1 + I_2 = m_1 r_1^2 + m_2 r_2^2$$

$$I = m_1 \frac{m_2^2 r^2}{(m_1 + m_2)^2} + m_2 \frac{m_1^2 r^2}{(m_1 + m_2)^2}$$

$$I = \frac{m_1 m_2 r^2}{(m_1 + m_2)} = \frac{m_1 m_2}{(m_1 + m_2)} r^2$$

5 (b)

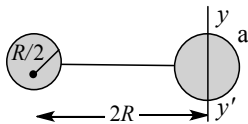
$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi \times \left[\frac{240}{60} - 0 \right]}{10}$$

$$\therefore \alpha = 2.51 \text{ rad/s}$$

6 (a)

$$\alpha = \frac{\omega}{t} = \frac{2\pi n}{t} = \frac{2\pi \left(\frac{540}{60} \right)}{6} = 3\pi \text{ rad/s}^2$$

7 (a)



Moment of inertia of the system about yy'

$I_{yy'}$ = Moment of inertia of sphere P about

yy' + Moment of inertia of sphere Q about yy'

Moment of inertia of sphere P about yy''

$$= \frac{2}{5} M \left(\frac{R}{2} \right)^2 + M(x)^2$$

$$= \frac{2}{5} M \left(\frac{R}{2} \right)^2 + M(2R)^2$$

$$= \frac{MR^2}{10} + 4MR^2$$

Moment of inertia of sphere Q about yy'' is $\frac{2}{5} M \left(\frac{R}{2} \right)^2$

$$\text{Now, } I_{yy'} = \frac{MR^2}{10} + 4MR^2 + \frac{2}{5} M \left(\frac{R}{2} \right)^2 = \frac{21}{5} MR^2$$

8 (b)

$$I = mR^2 = m \left(\frac{D^2}{4} \right) \Rightarrow I \propto mD^2 \text{ or } m \propto \frac{I}{D^2}$$

$$\therefore \frac{m_1}{m_2} = \frac{I_1}{I_2} \times \left(\frac{D_2}{D_1}\right)^2 = \frac{2}{1} \left(\frac{1}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$$

10 (a)

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -\hat{j} - 2\hat{k}$$

and the X - axis is given by $\hat{i} + 0\hat{j} + 0\hat{k}$

Dot product of these two vectors is zero *i.e.* angular momentum is perpendicular to X - axis

11 (c)

The rolling sphere has rotational as well as translational kinetic energy

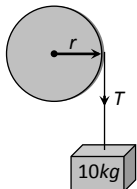
$$\begin{aligned} \therefore \text{Kinetic energy} &= \frac{1}{2}mu^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}mu^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = \frac{1}{2}mu^2 + \frac{mu^2}{5} = \frac{7}{10}mu^2 \end{aligned}$$

Potential energy = kinetic energy

$$\therefore mgh = \frac{7}{10}mu^2 \Rightarrow h = \frac{7u^2}{10g}$$

12 (d)

$$\begin{aligned} \tau &= r \times F \\ &= r \times T \\ &= r \times m \times g \\ &= 0.1 \times 10 \times 9.8 \\ &= 9.8 \text{ N-m} \end{aligned}$$



13 (d)

The moment of inertia of a circular disc

$$I = \frac{1}{2}MR^2$$

According to theorem of parallel axes

$$I' = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2 = 3I$$

14 (b)

Rotational kinetic energy = $\frac{1}{2}I\omega^2$

$$\therefore \text{Rotational KE} = \frac{1}{2}\left[\frac{1}{2}mr^2\right]_{r^2} \frac{v^2}{r^2} \quad (\text{where } I = \frac{1}{2}mr^2)$$

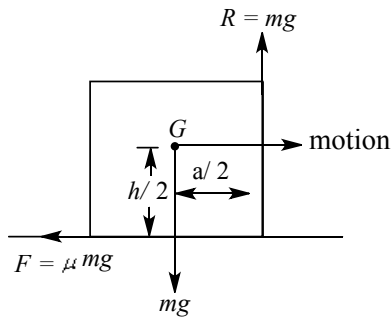
$$\begin{aligned} &4 = \frac{1}{2}\left[\frac{1}{2}(2)r^2\right]_{r^2} \frac{v^2}{r^2} \\ \Rightarrow &v^2 = 8 \end{aligned}$$

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$$v = 2\sqrt{2} \text{ ms}^{-1}$$

16 (b)

As shown in figure normal reaction $R = mg$. Frictional force $F = \mu R = \mu mg$. To topple, clockwise moment must be more than the anticlockwise moment i.e., $\mu mg \times \frac{h}{2} > mg \times \frac{a}{2}$ $\times \frac{a}{2}$ or $\mu > a/h$



18 (c)

Angle turned in three second, $\theta_{3s} = 2\pi \times 10 = 20\pi \text{ rad}$

$$\text{From } \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow 20\pi = 0 + \frac{1}{2} \alpha \times (3)^2$$

$$\Rightarrow \alpha = \frac{40\pi}{9} \text{ rad/s}^2$$

Now angle turned in 6 sec from the starting

$$\theta_{6s} = \omega_0 t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \times \left(\frac{40\pi}{9}\right) \times (6)^2 = 80\pi \text{ rad}$$

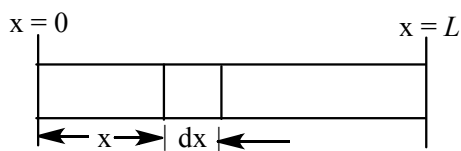
\therefore angle turned between $t = 3s$ to $t = 6s$

$$\theta_{\text{last } 3s} = \theta_{6s} - \theta_{3s} = 80\pi - 20\pi = 60\pi$$

$$\text{Number of revolution} = \frac{60\pi}{2\pi} = 30 \text{ rev}$$

19 (a)

$$x_{\text{CM}} = \frac{\int x dm}{\int dm}$$



If $n = 0$

$$\text{Then } x_{\text{CM}} = \frac{L}{2}$$

As n increases, the centre of mass shift away from $x = \frac{L}{2}$ which only option (a) is satisfying.

Alternately, you can use basic concept.

$$\begin{aligned} x_{\text{CM}} &= \frac{\int_0^L k \left(\frac{x}{L}\right)^n \times x dx}{\int_0^L k \left(\frac{x}{L}\right)^n dx} \\ &= L \left[\frac{n+1}{n+2} \right] \end{aligned}$$

20 (d)

$$\vec{v}_{\text{cm}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \frac{2 \times 3 + 3 \times 2}{2 + 3} = \frac{12}{5} = 2.4 \text{ m/s}$$

| ANSWER-KEY | | | | | | | | | | |
|-------------------|----|----|----|----|----|----|----|----|----|----|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A. | D | A | D | D | B | A | B | B | B | A |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| A. | C | D | D | B | A | B | C | C | A | D |
| | | | | | | | | | | |

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