CLASS : XITH
Solutions

## TOpic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1
(a)

In explosion of a bomb, only kinetic energy, changes as initial kinetic energy is zero
(d)

Solid and hollow balls can be distinguished by any of the three methods. $I_{\mathrm{h}}>I_{s}$. When torques are equal, angular acceleration $\alpha$ of hollow must be smaller than $\alpha$ of solid. Similarly, on rolling, solid ball will reach the bottom before the hollow ball
(b)

The acceleration of the body which is rolling down an inclined plane of angle $\alpha$ is


$$
a=\frac{\mathrm{g} \sin \alpha}{1+\frac{K_{2}}{R^{2}}}
$$

where $K=$ radius of gyration,
$R=$ radius of body.
Now, here the body is a uniform solid disc.
So, $\quad \frac{K^{2}}{R^{2}}=\frac{1}{2}$
$\therefore \quad a=\frac{\mathrm{g} \sin \alpha}{1+\frac{1}{2}}$
or $\quad a=\frac{\mathrm{g} \sin \alpha}{3 / 2}$
or $\quad a=\frac{2 \mathrm{~g} \sin \alpha}{3}$
(d)

Radius of gyration of circular disc $k_{\text {disc }}=\frac{R}{\sqrt{2}}$
Radius of gyration of circular ring $k_{\text {ring }}=R$
Ratio $=\frac{k_{\text {disc }}}{k_{\text {ring }}}=\frac{1}{\sqrt{2}}$
(c)

There is a point in the system, where if whole mass of the system is supposed to be concentrated, the nature of the motion executed by the system remains unaltered when the force acting on the system are applied directly at this point.
The position of centre of mass of system for $n$ particles is expressed as
or $\quad \sum m_{i} r_{i}=$ constant
Hence, for a system having particles, we have

$$
\begin{aligned}
& m_{1} r_{1}=m_{2} r_{2} \\
\Rightarrow \quad & \frac{r_{1}}{r_{2}}=\frac{m_{2}}{m_{1}}
\end{aligned}
$$

$i e$, the centre of mass of a system of two particle divides the distance between them in inverse ratio of masses of particles.
(d)

Angular momentum, $L=m v r=m \omega r^{2}=m \times \frac{2 \pi}{T} \times r^{2}$

$$
=\frac{2 \times 3.14 \times 6 \times 10^{24} \times\left(1.5 \times 10^{11}\right)^{2}}{3.14 \times 10^{7}}=2.7 \times 10^{40} \mathrm{~kg}-\mathrm{m}^{2} / \mathrm{s}
$$

(d)

Change in momentum $=\overrightarrow{\mathrm{F}}$ t and does not depend on mass of the bodies.
(c)

From the triangle $B C D$
$C D^{2}=B C^{2}-B D^{2}=a^{2}-\left(\frac{a}{2}\right)^{2}$
$x^{2}=\frac{3 a^{2}}{4} \Rightarrow x=\frac{\sqrt{3} a}{2}$


Moment of inertia of system along the side $A B$
$I_{\text {system }}=I_{1}+I_{2}+I_{3}=m \times(0)^{2}+m \times(x)^{2}+m \times(0)^{2}$

$$
=m x^{2}=m\left(\frac{\sqrt{3} a}{2}\right)^{2}=\frac{3 m a^{2}}{4}
$$

(a)

The moment of inertia of ring $=M R^{2}$
The moment of inertia of removed sector $=\frac{1}{4} M R^{2}$
The moment of inertia of remaining part $=M R^{2}-\frac{1}{4} M R^{2}$

$$
=\frac{3}{4} M R^{2}
$$

According to question, the moment of inertia of the remaining part $=k M R^{2}$
then, $\quad k=\frac{3}{4}$
(a)
$\frac{M L^{2}}{12}=M K^{2} \Rightarrow K=\frac{L}{\sqrt{12}}$
(c)
$I=m_{1} r_{1}^{2}+m_{2} r_{2}^{2}=2(0.3)^{2}+1(0.3)^{2}=0.27 \mathrm{~kg} \mathrm{~m}^{2}$
(a)

As two solid spheres are equal in masses, so

$$
\begin{array}{cc} 
& m_{A}=m_{B} \\
\Rightarrow & \frac{4}{3} \pi R_{A}^{3} \rho_{A}=\frac{4}{3} \pi R_{B}^{3} \rho_{B} \\
\Rightarrow & \frac{R_{A}}{R_{B}}=\left(\frac{\rho_{B}}{\rho_{A}}\right)^{1 / 3}
\end{array}
$$

The moment of inertia of sphere about diameter

$$
\begin{array}{rlrl} 
& & I=\frac{2}{5} m R^{2} \\
\Rightarrow \quad & \frac{I_{A}}{I_{B}}=\left(\frac{R_{A}}{R_{B}}\right)^{2} \quad\left(\text { as } m_{A}=m_{B}\right) \\
\Rightarrow \quad & \frac{I_{A}}{I_{B}}=\left(\frac{\rho_{B}}{\rho_{A}}\right)^{2 / 3}
\end{array}
$$

(d)

Linear kinetic energy $=\frac{1}{2} m v^{2}$
Rotational kinetic energy $=\frac{1}{2} I \omega^{2}$

$$
\begin{aligned}
& =\frac{1}{2}\left(\frac{2}{5} m r^{2}\right) \frac{v^{2}}{r^{2}} \\
& =\frac{1}{5} m v^{2} \\
& =\frac{1}{2} m v^{2}+\frac{1}{5} m \\
& =\frac{7}{10} m v^{2}
\end{aligned}
$$

$$
\text { Total KE }=\frac{1}{2} m v^{2}+\frac{1}{5} m v^{2}
$$

Required fraction $=\frac{\frac{1}{5} m v^{2}}{\frac{7}{10} m v^{2}}$

$$
=\frac{1}{5} \times \frac{10}{7}=\frac{2}{7}
$$

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(a)

Moment of inertia of big drop is $I=\frac{2}{5} M R^{2}$. When small droplets are formed from big drop volume of liquid remain same

$$
\begin{array}{lc} 
& n^{\frac{4}{3}} \pi r^{3}=\frac{4}{3} \pi R^{3} \\
\Rightarrow & n^{1 / 3} r=R \\
\text { as } & n=8 \\
\Rightarrow & r=\frac{R}{2}
\end{array}
$$

Mass of each small droplet $=\frac{M}{8}$
$\therefore$ Moment of inertia of each small droplet

$$
\begin{aligned}
& =\frac{2}{5}\left[\frac{M}{8}\right]\left[\frac{R}{2}\right]^{2} \\
& =\frac{1}{32}\left[\frac{2}{5} M R^{2}\right]=\frac{I}{32}
\end{aligned}
$$

(b)
$\frac{L_{\text {Total }}}{L_{B}}=\frac{\left(I_{A}+I_{B}\right) \omega}{I_{B} \cdot \omega} \quad$ (as $\omega$ will be same in both cases)

$$
\begin{array}{lr}
=\frac{I_{A}}{I_{B}}+1=\frac{m_{A} r_{A}^{2}}{m_{B} r_{B}^{2}}+1 & \\
=\frac{r_{A}}{r_{B}}+1 & \text { (as } \left.m_{A} r_{A}=m_{B} r_{B}\right) \\
=\frac{11}{2.2}+1 & \left(\text { as } r \propto \frac{1}{m}\right) \\
=6 &
\end{array}
$$

$\therefore$ The correct answer is 6 .
(b)

Moment of inertia of a rod about one end $=\frac{M L^{2}}{3}$
As, $I=I_{1}+I_{2}+I_{3}$
$\therefore I=0+\frac{M L^{2}}{3}+\frac{M L^{2}}{3}=\frac{2 M L^{2}}{3}$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| A. | A | D | B | A | B | A | D | C | D | D |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |
| A. | C | A | A | C | A | D | A | A | B | B |  |
|  |  |  |  |  |  |  |  |  |  |  |  |



