DPP

DAILY PRACTICE PROBLEMS

CLASS : XITH
DATE :

Solutions

SUBJECT : PHYSICS
DPP NO. : 7

Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1 **(b)**

In pure rolling, mechanical energy remains conserved. Therefore, when heights of inclines are equal, speed of sphere will be same in both the cases. But as acceleration down the incline, $a \propto \sin\theta$ therefore, acceleration and time of descent will be different

2 **(d)**

The compression of spring is maximum when velocities of both blocks A and B is same. Let it be v_0, then from conservation law of momentum

$mv = mv_0 + mv_0 = 2mv_0 \Rightarrow v_0 = \frac{v}{2}$

$\therefore$ kinetic energy of A - B system at that stage

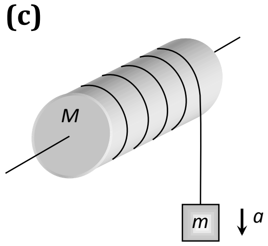
$= \frac{1}{2}(m+m) \times \left(\frac{v}{2}\right)^2 = \frac{mv^2}{4}$

Further loss in KE> = gain in elastic potential energy

ie, $\frac{1}{2}mv^2 - \frac{1}{4}mv^2 = \frac{1}{4}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow x = v\sqrt{\frac{m}{2k}}$

3 **(c)**



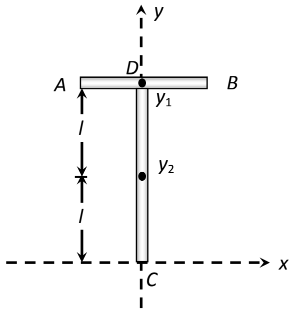
$a = \frac{g}{1+\frac{K^2}{R^2}}$ [For solid cylinder $\frac{K^2}{R^2} = \frac{1}{2}$]

$$\therefore a = \frac{g}{1+\frac{1}{2}} = \frac{2}{3}g$$

4 **(a)**

For translatory motion the force should be applied on the centre of the mass of the body.
So we have to calculate the location of centre of mass of 'T' shaped object.
Let mass of rod AB is m so the mass of the rod CD will be $2m$

Let y_1 is the centre of mass of rod AB and y_2 is the centre of mass of rod CD. We can consider that whole mass of the rod is placed at their respective centre of mass *i.e.*, mass m is placed at y_1 and mass $2m$ is placed at y_2



Taking point 'C' at the origin position vector of point y_1 and y_2 can be written as $\overrightarrow{r_1} = 2l\,\hat{j}$, $\overrightarrow{r_2} = l\hat{j}$ and $m_1 = m$ and $m_2 = 2m$

Position vector of centre of mass of the system

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\overrightarrow{r_2}}{m_1 + m_2} = \frac{m2l\hat{j} + 2ml\hat{j}}{m + 2m} = \frac{4ml\hat{j}}{3m} = \frac{4}{3}l\hat{j}$$

Hence the distance of centre of mass from $C = \frac{4}{3}l$

5 **(a)**

According to the equation of motion of the centre of mass

$M\,\mathbf{a}_{\mathrm{CM}} = \mathbf{F}_{\mathrm{ext}}$

If $\mathbf{F}_{\mathrm{ext}} = 0$, $\mathbf{a}_{\mathrm{CM}} = 0$

$\therefore\ \mathbf{v}_{\mathrm{CM}} = \text{constant}$

ie, if no external force acts on a system the velocity of its centre of mass remains constant. Thus, the centre of mass may move but not accelerate.

7 **(d)**

When a body rolls down an inclined plane, it is accompanied by rotational and translational kinetic energies.

Rotational kinetic energy $= \frac{1}{2}I\omega^2 = K_R$

Where I is moment of inertia and ω the angular velocity.

Translational kinetic energy

$$= \frac{1}{2}mv^2 = K_r = \frac{1}{2}m(r\omega)^2$$

where m is mass, v the velocity and ω the angular velocity.

Given,

Translational KE=rotational KE

$$\frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$$

Since, $v = r\omega$

$\therefore \quad \frac{1}{2}m(r^2\omega^2) = \frac{1}{2}I\omega^2$

$\Rightarrow \quad I = mr^2$

We know that mr^2 is the moment of inertia of hollow cylinder about its axis is where m is mass of hollow cylinderical body and r the radius of cylinder.

8 **(a)**

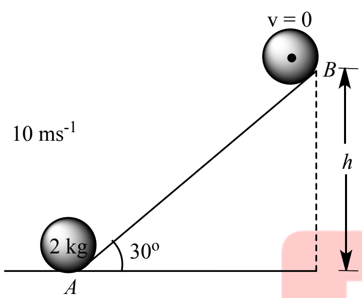
When a body rolls down without slipping along an inclined plane of inclination θ, it rotates about a horizontal axis through its centre of mass and also its centre of mass moves. Therefore, rolling motion may be regarded as a rotational motion about an axis through its centre of mass plus a translational motion of the centre of mass. As it rolls down, it suffers loss in gravitational potential energy provided translational energy due to frictional force is converted into rotational energy.

9 **(c)**

Given that,

Mass of solid sphere, $m = 2$ kg

Velocity, $v = 10$ ms^{-1}



Let the sphere attained a height h.

When the sphere is at point A, it possesses kinetic energy and rotational kinetic energy, and when it is at point B it possesses only potential energy.

So, from law of conservation of energy

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

or $\frac{1}{2}mv^2 + \frac{1}{2}mK^2 \times \frac{v^2}{R^2} = mgh$

or $\frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right] = mgh$

or $\frac{1}{2} \times (10)^2\left[1 + \frac{2}{5}\right] = 9.8 \times h$

(for solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$)

or $\frac{1}{2} \times 100 \times \frac{7}{5} = 9.8 \times h$

or $h = 7.1$ m

10 **(d)**

In the case of projectile motion, if bodies are projected with same speed, they reached at ground with same speeds. So, if bodies have same mass, then momentum of bodies or magnitude of momenta must be same

11 **(a)**

$$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}} = \frac{g \sin 30°}{1 + \frac{1}{2}} = \frac{g/2}{3/2} = \frac{g}{3}$$

12 **(c)**

This is an example of elastic oblique collision. When a moving body collides obliquely with another identical body in rest, then during elastic collision, the angle of divergence will be 90°

13 **(c)**

Angular retardation, $a = \frac{\tau}{I} = \frac{\mu\, mgR}{mR^2} = \frac{\mu g}{R}$

As, $\omega = \omega_0 - \alpha t$

$$\therefore \; t = \frac{\omega_0 - \omega}{\alpha} = \frac{\omega_0 - \omega_0/2}{\mu g/R} = \frac{\omega_0 R}{2\mu g}$$

14 **(d)**

$\frac{1}{2}mv^2 = \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$

$\therefore \quad I = \frac{1}{2}mR^2$

$\therefore$ Body is disc.

15 **(a)**

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 0.5 \times (0.1)^2 = 2.5 \times 10^{-3} kg - m^2$$

16 **(c)**

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{200 \times 10\hat{i} + 500 \times (3\hat{i} + 5\hat{j})}{200 + 500}$$

$$\vec{v}_{cm} = 5\hat{i} + \frac{25}{7}\hat{j}$$

17 **(c)**

Given, $I = \frac{2}{5}MR^2$

Using the theorem of parallel axes, moment of inertia of the sphere about a parallel axis tangential to the sphere is

$$I' = I + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\therefore \; I' = MK^2 = \frac{7}{5}MR^2, \qquad K = \left(\sqrt{\frac{7}{5}}\right)R$$

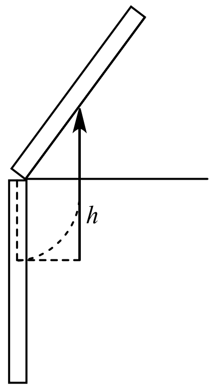
18 **(d)**

$TE_i = TE_r$

$\frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2} \times \frac{1}{3}ml^2\omega^2 = mgh$

$\Rightarrow \quad h = \frac{1}{6}\frac{l^2\omega^2}{g}$



19 **(b)**

The ratio of rotational kinetic energy to total kinetic energy is

$$\frac{KE_R}{KE_T}=\frac{\frac{1}{2}\,I\omega^2}{\frac{1}{2}I\omega^2+\frac{1}{2}\,Mv^2}$$

The moment of inertia of a ring

$$=\frac{1}{2}MR^2$$

$$\therefore \quad \frac{KE_R}{KE_T}=\frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left[\frac{v}{R}\right]^2}{\frac{1}{2}\times\frac{1}{2}MR^2\left[\frac{v}{R}\right]^2+\frac{1}{2}\,Mv^2}$$

$$=\frac{\frac{1}{2}\left(\frac{1}{2}Mv^2\right)}{\frac{1}{2}\left(\frac{1}{2}Mv^2\right)+\frac{1}{2}Mv^2}$$

$$\frac{KE_R}{KE_T}=\frac{1}{3}$$

20 **(c)**

From $t_1=0$ to $t_2=2t_0$ the external force acting on the combined system is $m_1\text{g}+m_2\text{g}$

$\therefore$ Total change in momentum of system

$=Ft=(m_1+m_2)\text{g}2t_0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	C	A	A	B	D	A	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	D	A	C	C	D	B	C