DPP
DAILY PRACTICE PROBLEMS

CLASS : XITH | **Solutions** | **SUBJECT : PHYSICS**
DATE : | | **DPP NO. : 6**

Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1 **(d)**

The angular momentum is measure for the amount of torque that has been applied over time the object. For a particle with a fixed mass that is rotating about a fixed symmetry axis, the angular momentum is expressed as

$$L = I\omega \quad \ldots(i)$$

Where I is moment of inertia of particle and ω the angular velocity.

Also, $K = \frac{1}{2}I\omega^2$ ...(ii)

Where K is kinetic energy of rotation.

From Eqs. (i) and (ii), we get

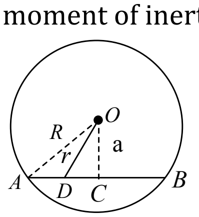
$$L = \frac{2K}{\omega^2}\omega = \frac{2K}{\omega}$$

$$L' = \frac{2(K/2)}{2\omega} = \frac{1}{4}\left(\frac{2K}{\omega}\right) = \frac{L}{4}$$

Note In a closed system angular momentum is constant.

2 **(c)**

As there is no external torque, angular momentum will remain constant. When the tortoise moves from A to C, figure, moment of inertia of the platform and tortoise decreases. Therefore, angular velocity of the system increases. When the tortoise moves from C to B, moment of inertia increases. Therefore, angular velocity decreases



If, M=mass of platform

R = radius of platform

m = mass of tortoise moving along the chord AB

a = perpendicular distance of O from AB

Initial angular momentum, $I_1 = mR^2 + \frac{MR^2}{2}$

At any time t, let the tortoise reach D moving with velocity v

$\therefore\ AD = vt$

$AC = \sqrt{R^2 - a^2}$

As $DC = AC - AD = (\sqrt{R^2 - a^2} - vt)$

$\therefore \; OD = r = a^2 + \left[\sqrt{R^2 - a^2} - vt\right]^2$

Angular momentum at time t

$$I_2 = mr^2 + \frac{MR^2}{2}$$

As angular momentum is conserved

$\therefore \; I_1\omega_0 = I_2\omega(t)$

This shows that variation of $\omega(t)$ with time is nonlinear. Choice (c) is correct

3 **(a)**

$$x_{CM} = \frac{m_1x_1 + m_2 + x_2 + \dots}{m_1 + m_2 + \dots}$$

$$= \frac{ml + 2m.2l + 3m.3l + \dots}{m + 2m + 3m + \dots}$$

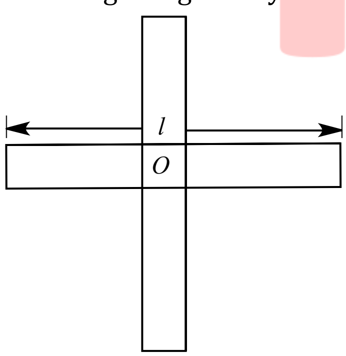
$$= \frac{ml(1 + 4 + 9 + \dots)}{m(1 + 2 + 3 + \dots)} = \frac{\frac{l\,n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{l(2n+1)}{3}$$

4 **(a)**

In the absence of external torque angular momentum remains constant

5 **(b)**

When l is length of rod and its mass, then the moment of inertia of its axis passing through its centre of gravity and perpendicular to its length is given by



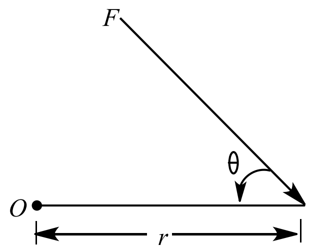
$$I = \frac{ml^2}{12}$$

For two rods as shown,

$$I = \frac{ml^2}{12} + \frac{ml^2}{12} = \frac{ml^2}{6}$$

6 **(d)**

Torque is a measure of how much a force acting on an object causes that object to rotate. The object rotates about an axis (O). The distance from O is r, where forces acts, hence torque $\tau = \mathbf{F} \times \mathbf{r}$. It is a vector quantity and points from axis of rotation to the point where the force acts.

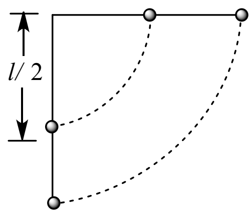


7 **(a)**

As the mass is concentrated at the centre of the rod, therefore,

$$mg \times \frac{l}{2} = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

or $l^2\omega^2 = 3gl$



Velocity of other end of the rod

$v = l\omega = \sqrt{3gl}$

8 **(d)**

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}} = \sqrt{\frac{2 \times 10 \times 3}{1 + \frac{mr^2}{2 \times mr^2}}} = \sqrt{\frac{2 \times 10 \times 3}{\frac{3}{2}}} = \sqrt{40}$$

$$\Rightarrow v = r\omega$$

$$\Rightarrow r = \frac{v}{\omega} = \frac{\sqrt{40}}{2\sqrt{2}} = \sqrt{\frac{40}{8}} = \sqrt{5}\, m$$

9 **(a)**

When non-conservative force acts on a body, then mechanical energy is converted into non-mechanical energy and *vice - versa*.

10 **(a)**

Work done = Change in rotational kinetic energy

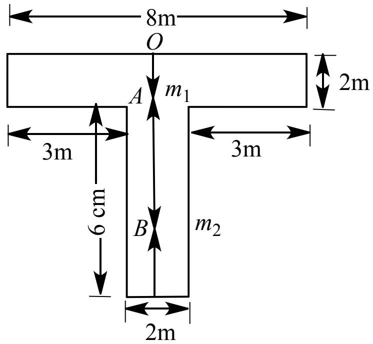
$$= \frac{1}{2} I \times (\omega_1^2 - \omega_2^2) = \frac{1}{2} I \times 4\pi^2(n_1^2 - n_2^2)$$

$$= \frac{1}{2} \times \frac{9.8}{\pi^2} \times 4\pi^2(10^2 - 5^2) = 9.8 \times 2 \times 75 = 1470\, J$$

11 **(b)**

Co-ordinate of CM is given by

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$



Taking parts A and B as two bodies of same system

$$m_1 = l \times b \times \sigma = 8 \times 2 \times \sigma = 16\sigma$$

$$m_2 = l \times b \times \sigma = 6 \times 2 \times \sigma = 12\sigma$$

Choosing O as origin,

$$x_1 = 1 \text{ m}, x_2 = 2 + 3 = 5 \text{ m}$$

$$\therefore \quad X_{CM} = \frac{16\sigma \times 1 + 12\sigma \times 5}{16\sigma + 12\sigma} = \frac{19}{7}$$

$$= 2.7 \text{ m from } O$$

12 **(b)**

Moment of inertia of a circular ring about a diameter

$$I = \frac{1}{2} Mr^2$$

13 **(b)**

The kinetic energy of a rolling body is

$$\frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2}\right)$$

According to law of conservation of energy, we get

$$\frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2}\right) = mg\text{h}$$

Where h is the height of the inclined plane

$$\therefore v = \sqrt{\frac{2g\text{h}}{1 + \frac{k^2}{R^2}}}$$

For a solid sphere $\frac{k^2}{R^2} = \frac{2}{5}$

Substituting the given values, we get

$$v = \sqrt{\frac{2 \times 10 \times 7}{\left(1 + \frac{2}{5}\right)}} = \sqrt{\frac{2 \times 10 \times 7 \times 5}{7}} = 10 \ ms^{-1}$$

14 **(d)**

M.I. of disc $I = \frac{1}{2} MR^2 = \frac{1}{2} M\left(\frac{M}{\pi \rho t}\right) = \frac{1}{2} \frac{M^2}{\pi \rho t}$

$\left(\text{As } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\pi R^2 t} \text{ therefore } R^2 = \frac{M}{\pi \rho t}\right)$

$\therefore I \propto \frac{1}{\rho}$ [If M and t are constant) $\Rightarrow \frac{I_1}{I_2} = \frac{\rho_2}{\rho_1}$

15 **(c)**

$\frac{1}{2} I\omega^2 = 360 \Rightarrow I = \frac{2 \times 360}{(30)^2} = \frac{2 \times 360}{30 \times 30} = 0.8 kg \times m^2$

16 **(d)**

(a) Impulsive received by m

$\vec{J} = m(\vec{v}_f - \vec{v}_i)$

$= m(-2\hat{i} + \hat{j} - 3\hat{i} - 2\hat{j})$

$= m(-5\hat{i} - \hat{j})$

And impulse received by M

$= -\vec{J} = m(5\hat{i} + \hat{j})$

(b) $mv = m(5\hat{i} + \hat{j})$

Or $v = \frac{m}{M}(5\hat{i} + \hat{j}) = \frac{1}{13}(5\hat{i} + \hat{j})$

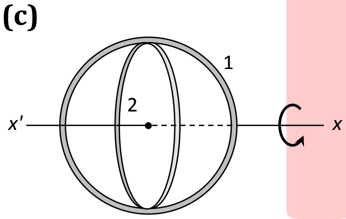
(c) $e =$ (relative velocity of separation/relative velocity of approach) in the direction of $-\vec{J}$

$= 11/17$

17 **(d)**

Remains conserved until the torque acting on it remain zero

18 **(c)**



$I_1 =$ M.I. of ring about its diameter $= \frac{1}{2} mr^2$

$I_2 =$ M.I. of ring about the axis normal to plane and passing through centre $= mr^2$

Two rings are placed according to figure. Then

$$I_{xx'} = I_1 + I_2 = \frac{1}{2} mr^2 + mR^2 = \frac{3}{2} mr^2$$

19 **(c)**

$\tau = \frac{dL}{dt}$, if $\tau = 0$ then $L =$ constant

20 **(b)**

If rod is rotated about end A, then vertical component of velocity $v_\perp$ of end A will be zero.

$\therefore \quad \omega = \frac{v \cos 60°}{l} = \frac{\sqrt{3} v}{2l}$

$= \frac{\sqrt{3} \times 3}{2 \times 0.5} = 5.2$ rads^{-1}

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	A	B	D	A	D	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	D	C	D	D	C	C	B