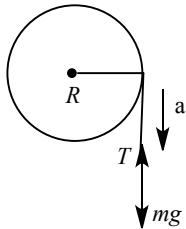


Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1 (d)

Let a be acceleration of fall of the thread, then net force acting downwards, balances the force due to tension (T) in the thread.



$$mg - T = ma$$

$$\Rightarrow mg - ma = T \quad \dots(i)$$

Also torque (also known as moment or couple acts on the system).

$\tau = \text{force} \times \text{perpendicular distance axis of rotation}$

$$\tau = T \times R$$

From Eq. (i),

$$\tau = m(g - a) \times R \quad \dots(ii)$$

Let I is moment of inertia of reel and α the angular acceleration, then torque is

$$\tau = I\alpha \quad \dots(iii)$$

where, $I = \frac{1}{2}MR^2$, $\alpha = \frac{a}{R}$

$$\therefore \tau = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{MRa}{2} \quad \dots(iv)$$

Equating Eqs. (ii) and (iv), we get

$$\tau = m(g - a)R = \frac{mRa}{2}$$

$$\Rightarrow g - a = \frac{a}{2}$$

$$\Rightarrow a = \frac{2}{3}g$$

2 (c)

Force of attraction between two stars

$$F = \frac{Gm_1m_2}{(r_1 + r_2)^2}$$

$$\text{Acceleration} = \frac{F}{m_1} = \frac{Gm_2}{(r_1 + r_2)^2}$$

3 **(c)**

Here, $m_1 = m_2 = 0.1 \text{ kg}$

$$r_1 = r_2 = 10 \text{ cm} = 0.1 \text{ m}$$

$$\begin{aligned} I &= I_1 + I_2 = m_1 r_1^2 + \frac{1}{2} m_2 r_2^2 = \frac{3}{2} m_1 r_1^2 \\ &= \frac{3}{2} \times 0.1 (0.1)^2 = 1.5 \times 10^{-3} \text{ kg m}^2 \end{aligned}$$

4 **(a)**

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

For disc and solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

As $\frac{K^2}{R^2}$ for solid sphere is smallest, it takes minimum time to reach the bottom of the incline

5 **(a)**

$$I = \frac{1}{2} MR^2 = \frac{1}{2} \times (\pi R^2 t \times \rho) \times R^2$$

$\Rightarrow I \propto R^4$ (As t and ρ are same)

$$\therefore \frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^4 = \left(\frac{0.2}{0.6}\right)^4 = \frac{1}{81}$$

6 **(c)**

$$m_1 r_1 = m_2 r_2$$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1} \therefore r \propto \frac{1}{m}$$

7 **(d)**

According to law of conservation of angular momentum, if there is no torque on the system, then the angular momentum remains constant.

8 **(b)**

Let the mass of an element of length dx of rod located at a distance x away from left end is $\frac{M}{L} dx$. The x -coordinate of the centre of mass is given by

$$\begin{aligned} X_{\text{CM}} &= \frac{1}{M} \int x \, dm \\ &= \frac{1}{M} \int_0^L x \left(\frac{M}{L} dx\right) \\ &= \frac{1}{L} \left[\frac{x^2}{2}\right]_0^L = \frac{L}{2} \end{aligned}$$

9 **(c)**

As $\vec{F}_{\text{ext}} = 0$, hence momentum remains conserved and final momentum = initial momentum = mv

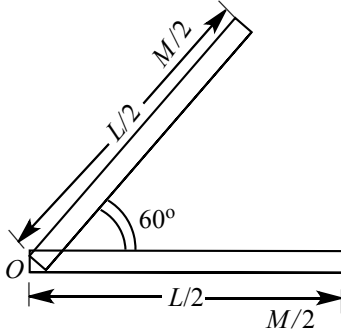
10 **(a)**

The moment of inertia of this annular disc about the axis perpendicular to its plane will be

11 $\frac{1}{2}M(R^2 + r^2).$

(b)

Since, rod is bent at the middle, so each part of it will have same length $(\frac{L}{2})$ and mass $(\frac{M}{2})$ as shown.



Moment of inertia of each part through its one end

$$= \frac{1}{3}(\frac{M}{2})(\frac{L}{2})^2$$

Hence, net moment of inertia through its middle point O is

$$I = \frac{1}{3}(\frac{M}{2})(\frac{L}{2})^2 + \frac{1}{3}(\frac{M}{2})(\frac{L}{2})^2$$

$$= \frac{1}{3}[\frac{ML^2}{8} + \frac{ML^2}{8}] = \frac{ML^2}{12}$$

12 **(c)**

$$K = \frac{L^2}{2I} = \frac{K_1}{K_2} = \frac{L_1^2}{L_2^2} \Rightarrow \frac{K_1}{K_2} = \left(\frac{100}{110}\right)^2 = \frac{100}{121}$$

$$\Rightarrow \frac{100}{K^2} = \frac{100}{121} \Rightarrow K_2 = 121 = 100 + 21$$

Increase in kinetic energy = 21%

13 **(c)**

$$I_{\text{Sphere}} < I_{\text{Disc}} < I_{\text{Shell}} < I_{\text{Ring}}$$

We know that body possessing minimum moment of inertia will reach the bottom first and the body possessing maximum moment of inertia will reach the bottom at last

14 **(a)**

Let a plane be inclined at an angle θ and a cylinder rolls down then the acceleration of the cylinder of mass m , radius R , and I

I as moment of inertia is given by

$$a = \frac{g \sin \theta}{\left(1 + \frac{I}{mR^2}\right)}$$

Moment of inertia (I) of a cylinder $= \frac{mR^2}{2}$

$$\therefore a = \frac{g \sin \theta}{\left(1 + \frac{\frac{mR^2}{2}}{mR^2}\right)} = \frac{2}{3}g \sin 30^\circ$$

$$\Rightarrow a = \frac{g}{3}$$

15 **(a)**

The rotational kinetic energy of the disc is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}MR^2\omega^2$$

The translational kinetic energy is

$$K_{\text{trans}} = \frac{1}{2}Mv_{\text{CM}}^2$$

where v_{CM} is the linear velocity of its centre of mass.

Now, $v_{\text{CM}} = R\omega$

Therefore, $K_{\text{trans}} = \frac{1}{2}MR^2\omega^2$

Thus, $K_{\text{total}} = \frac{1}{4}MR^2\omega^2 + \frac{1}{2}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2$

$$\therefore \frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{1}{4}MR^2\omega^2}{\frac{3}{4}MR^2\omega^2} = \frac{1}{3}$$

16 (c)

$$L = I\omega$$

17 (c)

Since force is not acting on centre of mass, it will produced torque hence linear and angular acceleration both will change

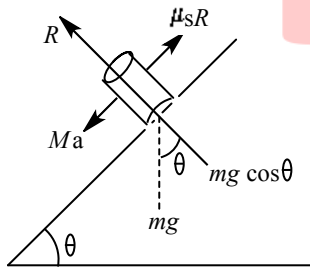
18 (a)

$$\begin{aligned} mgh &= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \\ &= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mv^2 = \frac{7}{10}mv^2 \end{aligned}$$

$$\therefore v = \sqrt{\frac{10}{7}gh}$$

19 (c)

For the rolling a solid cylinder acceleration $a = \frac{g}{3}\sin\theta$



\therefore The condition for the cylinder to remain in equilibrium

$$Ma \leq \mu_s R$$

$$\Rightarrow \frac{1}{2}Mg\sin\theta \leq Mg \cos\theta \cdot \mu_s$$

$$\text{or } \mu_s \geq \frac{1}{3}\tan\theta$$

$$\text{or } \tan\theta \leq 3\mu_s$$

20 (d)

Since, no external force is present on the system so, conservation principle of momentum is applicable

$$\therefore \vec{p}_i = \vec{p}_f = \vec{p}_1 + \vec{p}_2$$

$$\therefore \vec{p}_1 = -\vec{p}_2 \quad (\because \vec{p}_i = 0)$$

$$\therefore |\vec{p}_1| = |-\vec{p}_2|$$

$$\therefore \vec{p}_1 = \vec{p}_2$$

From this point of view, it is clear that momenta of both particles are equal in magnitude but opposite in direction

Also, friction is absent. So total mechanical energy of system remains conserved

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	C	A	A	C	D	B	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	C	A	A	C	C	A	C	D

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