CLASS : XITh
Solutions

## TOpic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1
(d)
$I=\frac{2}{5} M R^{2} \therefore I \propto R^{2}$
This relation shows that graph between $I$ and $R$ will be parabola symmetric to $I$-axis
(b)

According to the theorem of perpendicular axes.

$$
\begin{gathered}
I_{A B}+I_{C D}=M R^{2} \\
I_{d}+I_{d}=I
\end{gathered}
$$

$$
\begin{aligned}
2 I_{d} & =I \\
I_{d} & =\frac{I}{2}
\end{aligned}
$$


where, $I_{d}=$ moment of inertia about diameter of the ring, $I=$ moment of inertia about axes passing through to the ring.
6
(a)
${ }_{5}^{2} M R^{2}=\frac{1}{2} M r^{2}+M r^{2}$
or $\frac{2}{5} M R^{2}=\frac{3}{2} M r^{2}$
$\therefore \quad r=\frac{2}{\sqrt{15}} R$
(b)
$\frac{I_{\text {Ring }}}{I_{\text {Disc }}}=\frac{M R^{2}}{1 / 2 M R^{2}}=2: 1$

$$
\left(\because I_{A B}=I_{C D}=I_{d}\right)
$$

(b)
(I) Moment of inertia of a cylinder about its centre and parallel to its length $=\frac{M R^{2}}{2}$

(I)

(II)
(II) Moment of inertia about its centre and perpendicular to its length $=M\left(\frac{L^{2}}{12}+\frac{R^{2}}{4}\right)$
$\frac{M L^{2}}{12}+\frac{M R^{2}}{4}=\frac{M R^{2}}{2}$
Or $L=\sqrt{3} R$
(b)

Let at the time explosion velocity of one piece of mass $m / 2$ is ( $10 \hat{i}$ ). If velocity of other be $\vec{v}_{2}$, then from conservation law of momentum (since there is no force in horizontal direction), horizontal component of $\vec{v}_{2}$, must be - $10 \hat{i}$.
$\therefore$ Relative velocity of two parts in horizontal direction $=20 \mathrm{~ms}^{-1}$
Time taken by ball to fall through 45 m ,
$=20=\sqrt{\frac{2 h}{\mathrm{~g}}}=\sqrt{\frac{2 \times 45}{10}}=3 \mathrm{~s}$ and time taken by ball to fall through first $20 \mathrm{~m}, t^{\prime}=\sqrt{\frac{2 h^{\prime}}{\mathrm{g}}}=$
$\sqrt{\frac{2 \times 20}{10}}=2 \mathrm{~s}$. Hence time taken by ball pieces to fall from 25 m height to ground $=t-t^{\prime}=3$
$-2=1 \mathrm{~s}$.
$\therefore$ Horizontal distance between the two pieces at the time of striking on ground
$=20 \times 1=20 \mathrm{~m}$
(c)

Graph should be parabola symmetric to $I$-axis, but it should not pass from origin because there is a constant value $I_{c m}$ is present for $x=0$
(d)

Weight of the rod will produce the torque
$\tau=I \alpha \Rightarrow m g \times \frac{l}{2}=\frac{m l^{2}}{3} \times \alpha$


Angular acceleration
$\alpha=\frac{3 g}{2 l}$
(a)

The situation can be shown as


Let radius of complete disc is $a$ and that of small disc is $b$. Also let centre of mass now shifts to $O_{2}$ at a distance $x_{2}$ from original centre.
The position of new centre of mass is given by

$$
X_{\mathrm{CM}}=\frac{-\sigma \pi b^{2} x_{1}}{\sigma \pi a^{2}-\sigma \pi b^{2}}
$$

Here, $a=6 \mathrm{~cm}, b=2 \mathrm{~cm}, x_{1}=3.2 \mathrm{~cm}$
Hence, $X_{\mathrm{CM}}=\frac{-\sigma \times \pi(2)^{2} \times 3.2}{\sigma \times \pi \times(6)^{2}-\sigma \times \pi \times(2)^{2}}$

$$
=-\frac{12.8 \pi}{32 \pi}=-0.4 \mathrm{~cm}
$$

(a)

Initial acceleration of the system is zero. So it will always remain zero because there is no external force on the system
(d)

Rotational kinetic energy $=\frac{1}{2} I \omega^{2}=\frac{1}{2}\left(\frac{1}{2} M R^{2}\right) \times \omega^{2}$
$=\frac{1}{2}\left(\frac{1}{2} \times 10 \times(0.5)^{2}\right) \times(20)^{2}=250 \mathrm{~J}$
(d)
$\frac{1}{2} I \omega^{2}=\frac{1}{2} m v^{2} \Rightarrow \frac{1}{2} \times 3 \times(2)^{2}=\frac{1}{2} \times 12 \times v^{2}$

$$
\Rightarrow v=1 \mathrm{~m} / \mathrm{s}
$$

(b)

In doing so moment of inertia is decreased and hence angular velocity is increased
16 (b)
In the absence external force, position of centre of mass remain same therefore they will meet at their centre of mass
(a)

Moment of inertia of $\operatorname{rod} A B$ about point $P$ and perpendicular to the plane $=\frac{M l^{2}}{12}$

M.I. of rod $A B$ about point ${ }^{\prime} O^{\prime}=\frac{M l^{2}}{12}+M\left(\frac{l}{2}\right)^{2}=\frac{M l^{2}}{3}$
(By using parallel axis theorem)

But the system consists of four rods of similar type so by but the symmetry $I_{\text {System }}=4\left(\frac{M l^{2}}{3}\right)$
(c)
$x_{C M}=\frac{\sum m_{i} x_{i}}{\sum m_{i}}$, Refer to figure
$=\frac{M \times 0+M \times 1+M \times 2}{M+M+M}=1$

$y_{C M}=\frac{\sum m_{i} y_{i}}{\sum m_{i}}$
$=\frac{M \times 0+M\left(2 \sin 60^{\circ}\right)+M \times 0}{M+M+M}$
$=\frac{\sqrt{3} M}{3 M}=\frac{1}{\sqrt{3}}$
$\therefore$ Position vector of centre of mass is $\left(\hat{\mathbf{i}}+\frac{1}{\sqrt{3}} \hat{\mathbf{j}}\right)$
(a)

Angular velocity $=\omega$
Centripetal force $F=m r \omega^{2}$
or

$$
r \propto \frac{1}{\omega^{2}}
$$

$\therefore \quad \frac{r_{1}}{r_{2}}=\frac{\omega_{2}^{2}}{\omega_{1}^{2}}$
or

$$
\frac{4}{r_{2}}=\frac{4 \omega^{2}}{\omega^{2}}
$$


or

$$
r_{2}=1 \mathrm{~cm}
$$

(b)

Time of descent will be less for solid sphere i.e. solid sphere will reach first at the bottom of inclined plane

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| A. | D | C | A | B | B | B | B | C | C | D |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |
| A. | A | A | D | D | D | B | A | C | A | B |  |
|  |  |  |  |  |  |  |  |  |  |  |  |



