CLASS : XITh
Solutions

## Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1
(b)

Here, Moment of inertia, $I=3 \times 10^{2} \mathrm{~kg} \mathrm{~m}^{2}$
Torque, $\tau=6.9 \times 10^{2} \mathrm{Nm}$
Initial angular speed, $\omega_{0}=4.6 \mathrm{rad} \mathrm{s}^{-1}$
Final angular speed, $\omega=0 \mathrm{rad} \mathrm{s}^{-1}$
As $\omega=\omega_{0}+\alpha t$

$$
\alpha=\frac{\omega-\omega_{0}}{t}=\frac{0-4.6}{t}=-\frac{4.6}{t} \mathrm{rad} \mathrm{~s}^{-2}
$$

Negative sign is for deceleration Torque, $\tau=I \alpha$

$$
\begin{gathered}
6.9 \times 10^{2}=3 \times 10^{2} \times \frac{4.6}{t} \\
t=\frac{3 \times 10^{2} \times 4.6}{6.9 \times 10^{2}}=2 \mathrm{~s}
\end{gathered}
$$

(d)

Here, mass of the disc, $M=1 \mathrm{~kg}$
Radius of the disc, $R=2 m$
Moment of inertia of the circular disc about $X Y$ is

$I_{X Y}=\frac{M R^{2}}{2}=2 \mathrm{~kg} \mathrm{~m}^{2} \quad$ [Given]
According to theorem of parallel axes the moment of inertia of the circular disc about $X_{1} Y_{1}$ is

$$
\begin{aligned}
& I_{X_{1} Y_{1}}=I_{X Y}+M R^{2}=\frac{M R^{2}}{2}+M R^{2} \\
& =\frac{3}{2} M R^{2}=\frac{3}{2} \times(1 \mathrm{~kg}) \times(2 \mathrm{~m})^{2}=6 \mathrm{kgm}^{2}
\end{aligned}
$$

(b)

According to the theorem of || axes, Moment of inertia of disc about an axis passing through $K$ and $\perp$ to plane of disc,

$=\frac{1}{2} M R^{2}+M R^{2}=\frac{3}{2} M R^{2}$
Total moment of inertia of the system
$=\frac{3}{2} M R^{2}+m(2 R)^{2}+m(\sqrt{2} R)^{2}+m(\sqrt{2} R)^{2}$
$=3(M+16 m) \frac{R^{2}}{12}$
(a)

$$
\frac{1}{2} m v^{2}\left(1+\frac{K^{2}}{R^{2}}\right)=\frac{1}{2}(0.5)(0.2)^{2}\left(1+\frac{2}{5}\right)=0.014 \mathrm{~J}
$$

(b)

$$
\begin{aligned}
& \frac{m a^{2}}{12}+m\left(4 a^{2}\right)=\frac{m a^{2}}{6}+m d^{2} \\
& \text { or } \quad d^{2}=\frac{47 a^{2}}{12} \\
& \therefore \quad d=\sqrt{\frac{47}{12}} a
\end{aligned}
$$

(a)

The resultant force on the system is zero. So, the centre of mass of system has no acceleration
(b)

According to law of conservation of momentum
$I \omega=$ constant
When viscous fluid of mass $m$ is dropped and start spreading out then its moment of inertia increases and angular velocity decreases. But when it falls from the platform moment of inertia decreases so angular velocity increases again
(d)

For disc, $I=\frac{1}{2} m a^{2}$
For ring, $I=m a^{2}$
For square of side $2 a=\frac{M}{12}\left[(2 a)^{2}+(2 a)^{2}\right]=\frac{2}{3} M a^{2}$
For square of rod of length $2 a$

$$
I=4\left[M \frac{(2 a)^{2}}{12}+M a^{2}\right]=\frac{16}{3} M a^{2}
$$

Hence, moment of inertia is maximum for square of four rods

9
(b)

M.I. of block about $x$ axis, $I_{x}=\frac{m}{12}\left(b^{2}+t^{2}\right)$
M.I. of block about $y$ axis, $I_{y}=\frac{m}{12}\left(l^{2}+t^{2}\right)$
M.I. of block about $z$ axis, $I_{z}=\frac{m}{12}\left(l^{2}+b^{2}\right)$

As $I>b>t \therefore I_{z}>I_{y}>I_{x}$
(a)

Mass of a rod, $m=1.8 \mathrm{~kg}$
$\therefore$ Weight of a rod, $W=m g=1.8 \mathrm{~kg} \times 10 \mathrm{~ms}^{-2}=18 \mathrm{~N}$


As the rod is uniform, therefore weight of the rod is acting at its midpoint Taking moments about $A$,

$$
27 \times \frac{L}{4}+18 \times \frac{L}{2}=F \times L
$$

$\Rightarrow F L=\frac{L}{4}[27+36]=\frac{63 L}{4} \Rightarrow F=\frac{63}{4}=16 \mathrm{~N}$
(b)

Given : kinetic energy $K=360 J$
Angular speed $\omega=20 \mathrm{rad} / \mathrm{s}$
$\therefore K=\frac{1}{2} I \omega^{2}$
Where $I=$ moment of inertia
$\Rightarrow I=\frac{2 K}{\omega^{2}}=\frac{2 \times 360}{20 \times 20}=1.8 \mathrm{Akg} \mathrm{m}^{-2}$
(a)
$\tau=\frac{d L}{d t}=\frac{L_{2}-L_{1}}{\Delta^{t}}=\frac{5 L-2 L}{3}=\frac{3 L}{3}=L$
(a)

By the theorem of perpendicular axes, the moment of inertia about the central axis $I_{C}$, will be equal to the sum of its moments of inertia about two mutually perpendicular diameters lying in its plane.

Thus, $\quad I_{d}=I=\frac{1}{2} M R^{2}$
$\therefore \quad I_{C}=I+I$

$$
=\frac{1}{2} M R^{2}+\frac{1}{2} M R^{2}
$$

$$
=I+I=2 I
$$

(d)
$L=\sqrt{2 I E}$. If $E$ are equal then $\frac{L_{1}}{L_{2}}=\sqrt{\frac{I_{1}}{I_{2}}}=\sqrt{\frac{I}{2 I}}=\frac{1}{\sqrt{2}}$
(c)

Frequency of wheel, $v=\frac{300}{60}=5 \mathrm{rps}$. Angle described by wheel in one rotation $=2 \pi \mathrm{rad}$
Therefore, angle described by wheel in 1 s
$=2 \pi \times 5 \mathrm{rad}$
$=10 \pi \mathrm{rad}$.
(a)
(1)Angular velocity of earth

$$
\begin{aligned}
\omega_{1} & =\frac{2 \pi}{T}=\frac{2 \pi}{24 \times 60 \times 60} \\
& =\frac{2 \pi}{86400} \mathrm{rads}^{-1}
\end{aligned}
$$

(2)Angular velocity of hour's hand of a clock

$$
\begin{aligned}
\omega_{2} & =\frac{2 \pi}{T} \\
& =\frac{2 \pi}{12 \times 60 \times 60} \\
& =\frac{2 \pi}{43200} \mathrm{rads}^{-1}
\end{aligned}
$$

(3)Angular velocity of seconds hand of a clock

$$
\begin{aligned}
\omega_{3} & =\frac{2 \pi}{T} \\
& =\frac{2 \pi}{1 \times 60}=\frac{2 \pi}{60} \mathrm{rads}^{-1}
\end{aligned}
$$

(4)Angular velocity of flywheel

$$
\begin{aligned}
\omega_{4} & =2 \pi n \\
& =2 \pi \times \frac{300}{60} \\
& =2 \pi \times 5 \mathrm{rads}^{-1}
\end{aligned}
$$

(b)

Let ball strikes at a speed $u$ the $K_{1}=\frac{1}{2} m u^{2}$
Due to collision tangential component of velocity remains unchanged at $u \sin 45^{\circ}$, but the normal component of velocity change to $u \sin 45^{\circ}=\frac{1}{2} u \cos 45^{\circ}$
$\therefore$ Final velocity of ball after collision
$v=\sqrt{\left(u \sin 45^{\circ}\right)^{2}+\left(\frac{1}{2} u \cos 45^{\circ}\right)^{2}}$
$=\sqrt{\left(\frac{u}{\sqrt{2}}\right)^{2}+\left(\frac{u}{2 \sqrt{2}}\right)^{2}}=\sqrt{\frac{5}{3} u .}$
Hence final kinetic energy $K_{2}=\frac{1}{2} m v^{2}=\frac{5}{16} m u^{2}$.
$\therefore$ Fractional loss in KE
$=\frac{K_{1}-K_{2}}{K_{1}}=\frac{\frac{1}{2} m u^{2}-\frac{5}{16} m u^{2}}{\frac{1}{2} m u^{2}}=\frac{3}{8}$
(a)

Clearly, the question refers to the torque about an axis through the centre of wheel. Then, since the radius to the point application of the force is the lever or momentum arm.
we have

$\tau=0.25 \times 200=50 \mathrm{Nm}$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| A. | B | D | B | A | B | A | B | D | B | A |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |
| A. | B | A | D | A | D | D | C | A | B | A |  |
|  |  |  |  |  |  |  |  |  |  |  |  |



