CLASS : XITh
Solutions

## Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1
(c)

Moment of inertia of uniform circular disc about diameter $=I$
According to theorem of perpendicular axes,
Moment of inertia of disc about its axis $=2 I\left(=\frac{1}{2} m r^{2}\right)$
Applying theorem of parallel axes
Moment of inertia of disc about the given axis

$$
=2 I+m r^{2}=2 I+4 I=6 I
$$

2

3
(d)

Angular displacement during time

$$
\begin{aligned}
\theta & =\left(\omega_{2}-\omega_{1}\right) t \\
& =\left(2 \pi n_{2}-2 \pi n_{1}\right) t \\
& =(600 \pi-200 \pi) \times 10 \\
& =4000 \pi \mathrm{rad}
\end{aligned}
$$

Therefore, number of revolutions made during this time

$$
=\frac{4000 \pi}{2 \pi}=2000
$$

(b)

Let rod is placed along $x$-axis. Mass of element $P Q$ of length $d x$ situated at $x=x$ is


$$
d m=\lambda d x=(2+x) d x
$$

The CM of the element has coordinates $(x, 0,0)$.
Therefore, $x$-coordinates of CM of the rod will be

$$
\begin{aligned}
x_{\mathrm{CM}}= & \frac{\int_{0}^{3} x d m}{\int_{0}^{3} d m} \\
& =\frac{\int_{0}^{3} x(2+x) d x}{\int_{0}^{3}(2+x) d x} \\
& =\frac{\int_{0}^{3}\left(2 x+x^{2}\right) d x}{\int_{0}^{3}(2+x) d x} \\
& =\frac{\left[\frac{2 x^{2}}{2}+\frac{x^{3}}{3}\right]_{0}^{3}}{\left[2 x+\frac{x^{2}}{2}\right]_{0}^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\left[(3)^{2}+\frac{(3)^{3}}{3}\right]}{\left[2 \times 3+\frac{(3)^{2}}{2}\right]}=\frac{9+9}{6+9 / 2} \\
& =\frac{18 \times 2}{21}=\frac{12}{7} \mathrm{~m}
\end{aligned}
$$

4
(d)

$$
I=M K^{2}=160 \Rightarrow K^{2}=\frac{160}{M}=\frac{160}{10}=16 \Rightarrow K=4 \text { metre }
$$

(b)

As the mass of disc is negligible therefore only moment of inertia of five particles will be considered

$$
I=\sum m r^{2}=5 m r^{2}=5 \times 2 \times(0.1)^{2}=0.1 k g-m^{2}
$$

(b)
$\tau=I \alpha$, if $\tau=0$ then $\alpha=0$ because moment of inertia of any body cannot be zero
(a)

Since, no external torque is acting the angular $(J)$ is conserved.
$J=I \omega=$ constant
Where $I\left(=m r^{2}\right)$ is moment of inertia and $\omega$ the angular velocity.
Given, $I_{1}=I, I_{2}=\frac{I}{n}, \omega_{1}=\omega$

$$
\begin{array}{cc}
\therefore & J=I_{1} \omega_{1}=I_{2} \omega_{2} \\
& I \omega=\frac{I}{n} \omega_{2} \\
\Rightarrow & \omega_{2}=n \omega
\end{array}
$$

Hence, angular velocity increases by a factor of $n$.
(c)

Loss of kinetic energy $=\frac{1}{2} \frac{m_{1} m_{2}}{m_{1}+m_{2}}\left(v_{1}-v_{2}\right)^{2}$

$$
\begin{aligned}
& =\frac{1}{2} \frac{M \times M}{(M+M)}\left(v_{1}-v_{2}\right)^{2} \\
& =\frac{M \cdot M}{2(2 M)}\left(v_{1}-v_{2}\right)^{2} \\
& =\frac{M}{4}\left(v_{1}-v_{2}\right)^{2}
\end{aligned}
$$

(b)

Angular moment of particle w.r.t., origin
$=$ linear momentum $\times$ perpendicular distance of line of action of linear momentum from origin

(a)

He decreases his Moment of inertia by this act and therefore increases his angular velocity
(c)

Let $T$ be the tension in the string carrying
The masses $m$ and $3 m$
Let $a$ be the acceleration, then

$$
\begin{align*}
& T-m \mathrm{~g}=m a  \tag{i}\\
& 3 m \mathrm{~g}-T=3 m a \tag{ii}
\end{align*}
$$

Adding Eqs. (i) and (ii), we het

(c)

Let same mass and same outer radii of solid sphere and hollow sphere are $M$ and $R$ respectively. The moment of inertia of solid sphere $A$ about its diameter

$$
\begin{equation*}
I_{A}=\frac{2}{5} M R^{2} \tag{i}
\end{equation*}
$$

Similarly, the moment of inertia of hollow sphere (spherical shell) $B$ about its diameter

$$
\begin{equation*}
I_{B}=\frac{2}{3} M R^{2} \tag{ii}
\end{equation*}
$$

It is clear from Eqs. (i) and (ii), we get

$$
I_{A}<I_{B}
$$

(a)

By the conservation of energy
P.E. of rod = Rotational K.E.

$m g \frac{l}{2} \sin \alpha=\frac{1}{2} I \omega^{2} \Rightarrow m g \frac{l}{2} \sin \alpha=\frac{1}{2} \frac{m l^{2}}{3} \omega^{2}$

$$
\Rightarrow \omega=\sqrt{\frac{3 g \sin \alpha}{l}}
$$

But in the problem length of the $\operatorname{rod} 2 L$ is given

$$
\Rightarrow \omega=\sqrt{\frac{3 g \sin \alpha}{2 L}}
$$

(c)

Time taken in reaching bottom of incline is
$t=\sqrt{\frac{2 l\left(1+K^{2} / R^{2}\right)}{\mathrm{g} \sin \theta}}$
For solid cylinder (SC), $K^{2}=R^{2} / 2$
For hollow cylinder (HC), $K^{2}=R^{2}$
For solid sphere (S), $K^{2}=\frac{2}{5} R^{2}$
(b)

According to figure let $A$ is the origin and co-ordinates of centre of mass be $(x, y)$ then,

$$
\begin{aligned}
& x=\frac{m_{1} x_{1}+m_{2} x_{2}+m_{3} x_{3}+m_{4} x_{4}}{m_{1}+m_{2}+m_{3}+m_{4}} \\
& =\frac{0+2 \times \frac{80}{\sqrt{2}}+4 \times \frac{80}{\sqrt{2}}+0}{16}=\frac{30}{\sqrt{2}}
\end{aligned}
$$

Similarly $y=\frac{30}{\sqrt{2}}$ so, $r=\sqrt{x^{2}+y^{2}}=30 \mathrm{~cm}$

(b)
$I=M K^{2}=\sum m R^{2}$
where $M$ is the total mass of the body.
This means that

$$
K=\sqrt{\left(\frac{I}{M}\right)}
$$

According to thermo of parallel axis

$$
I=I_{C G}+M(2 R)^{2}
$$


where, $I_{C G}$ is moment of inertia about an axis through centre of gravity.

$$
\begin{array}{ll}
\therefore & I=\frac{2}{5} M R^{2}+4 M R^{2}=\frac{22}{5} M R^{2} \\
\text { or } & M K^{2}=\frac{22}{5} M K^{2} \\
\therefore & K=\sqrt{\frac{22}{5}} R
\end{array}
$$

(a)

The situation can be shown as: Let radius of complete disc is $a$ and that of small disc is $b$, also let centre of mass now shifts to $O_{2}$ at a distance $x_{2}$ from original centre


The position of new centre of mass is given by

$$
X_{C M}=\frac{-\sigma \pi b^{2} \cdot x_{1}}{\sigma \pi a^{2}-\sigma . \pi b^{2}}
$$

Here, $a=6 \mathrm{~cm}, b=2 \mathrm{~cm} x_{1}=3.2 \mathrm{~cm}$
Hence, $X_{C M}=\frac{-\sigma \times \pi(2)^{2} \times 3.2}{\sigma \times \pi \times(6)^{2}-\sigma \times \pi \times(2)^{2}}$

$$
=\frac{12.8 \pi}{32 \pi}=-0.4 \mathrm{~cm}
$$

(b)

$$
v=\sqrt{\frac{2 g_{\mathrm{h}}}{1+\frac{K^{2}}{R^{2}}}}=\sqrt{\frac{2 g_{\mathrm{h}}}{1+\frac{1}{2}}}=\sqrt{\frac{4}{3} g_{\mathrm{h}}}
$$

(c)
$I=\frac{2}{5} M R^{2}=\frac{2}{5}\left(\frac{4}{3} \pi R^{3} \rho\right) R^{2}=\frac{8}{15} \times \frac{22}{7} R^{5} \rho$
$I=\frac{176}{105} R^{5} \rho$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A. | C | D | B | D | B | B | A | C | B | A |
|  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| A. | C | C | A | C | B | B | A | B | C | C |
|  |  |  |  |  |  |  |  |  |  |  |



