CLASS : XITH
Solutions

## Topic :- SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1
(c)
M.I. of the plate about an axis perpendicular to its plane and passing through its centre

$I_{0}=\frac{m a^{2}}{6}$
By parallel axes theorem
$I_{A}=I_{0}+m\left(\frac{a}{\sqrt{2}}\right)^{2}=\frac{2}{3} m a^{2}$
2
(c)

Moment of inertia of a disc

$$
I=\frac{1}{2} M R^{2}
$$

Disc is melted and recasted into a solid sphere.
$\therefore$ Volume of sphere=Volume of disc

$$
\begin{aligned}
& \frac{4}{3} \pi R_{1}^{3}=\pi R^{2} \times \frac{R}{6} \\
& \frac{4}{3} R_{1}^{3}=\frac{R^{3}}{6} \\
& R_{1}^{3}=\frac{R^{3}}{8} \quad \Rightarrow \quad R_{1}=\frac{R}{2}
\end{aligned}
$$

$\therefore$ Moment of inertia of sphere

$$
I^{\prime}=\frac{2}{5} M R_{1}^{2}=\frac{2}{5} M\left(\frac{R}{2}\right)^{2}=\frac{2}{5} \frac{2 R^{2}}{4}=\frac{1}{5}\left(\frac{1}{2} M R^{2}\right)=\frac{I}{5}
$$

3
(c)

For solid cylinder, $\theta=30^{\circ}, K^{2}=\frac{1}{2} R^{2}$
For hollow cylinder, $\theta=$ ?, $K^{2}=R^{2}$
Using we find,
$\frac{\left(1+\frac{1}{2}\right)}{\sin 30^{\circ}}=\frac{1+1}{\sin \theta}$
$\therefore \sin \theta=\frac{2}{3}=0.6667$
$\theta=42^{\circ}$
(c)

We can assume that three particles of equal mass $m$ are placed at the corners of triangle $\vec{r}_{1}=0 \hat{i}+0 \hat{j}, \vec{r}_{2}=b \hat{i}+0 \hat{j}$
and $\vec{r}_{3}=0 \hat{i}+h \hat{j}$
$\therefore \overrightarrow{r_{c m}}=\frac{m_{1} \vec{r}_{1}+m_{2} \vec{r}_{2}+m_{3} \vec{r}_{3}}{m_{1}+m_{2}+m_{3}}$
$(0, h) \stackrel{y}{c}$
$=\frac{b}{3} \hat{i}+\frac{\mathrm{h}}{3} \hat{j}$
i.e. coordinates of centre of mass is $\left(\frac{b}{3}, \frac{h}{3}\right)$
(d)

When a heavy body with velocity $u$ collides with a lighter body at rest, then the heavier body remains moving in the same direction with almost same velocity. The lighter body moves in the same direction with a nearly velocity of $2 u$
(b)

Since, no force is present along the surface of plane so, momentum conservation principle for ball is applicable along the surface of plate.

$m v \sin \theta_{1}=m v_{1} \sin \theta_{2}$
$m v \sin \theta_{1}=m v_{1} \sin \theta_{2}$
Or $v \sin \theta_{1}=v_{1} \sin \theta_{2}$
$e=\frac{v_{1} \cos \theta_{2}}{v \cos \theta_{1}}=\frac{v_{1} \cos \theta_{2}}{v \cos \theta}$
$\therefore v_{1} \cos \theta_{2}=e v \cos \theta$
$\therefore \frac{v_{1} \sin \theta_{2}}{v_{1} \cos \theta_{2}}=\frac{v \sin \theta}{e v \cos \theta}=\frac{\tan \theta}{e}$
$\therefore \tan \theta=\frac{\tan \theta}{e}$
$\therefore \theta_{2}=\tan ^{-1}\left(\frac{\tan \theta}{e}\right)$
(d)

We know that angular momentum of spin $=I \omega$
By the conservation of angular momentum

$$
\begin{array}{r}
\frac{2}{5} M R^{2} \cdot \frac{2 \pi}{T}=\frac{2}{5} M\left(\frac{R}{4}\right)^{2} \cdot \frac{2 \pi}{T^{\prime}} \\
T^{\prime}=\frac{T}{16}=\frac{24}{16}=1.5 \mathrm{~h}
\end{array}
$$

(d)

Melting of ice produces water which will spread over larger distance away from the axis of rotation. This increases the moment of inertia so angular velocity decreases

Hence, $m_{1}=10 \mathrm{~kg}, m_{2}=4 \mathrm{~kg}$
$v_{1}=14 \mathrm{~ms}^{-1}, v_{2}=0$
$v_{\mathrm{CM}}=\frac{m_{1} v_{1}+m_{2} v_{2}}{m_{1}+m_{2}}$
$v_{\mathrm{CM}}=\frac{10 \times 14+4 \times 0}{10+4}=10 \mathrm{~ms}^{-1}$
(b)

Let centre of mass of lead sphere after hollowing be at point $O_{2}$, where $\mathrm{OO}_{2}=x$
Mass of spherical hollow $m=\frac{\frac{4}{3} \pi\left(\frac{R}{2}\right)^{2} M}{\left(\frac{4}{2} \pi R^{3}\right)}=\frac{M}{8}$ and
$x=O O_{1}=\frac{R}{2}$

$\therefore x=\frac{M \times 0-\left(\frac{M}{8}\right) \times \frac{R}{2}}{M-\frac{M}{8}}=\frac{\frac{M R}{16}}{\frac{7 M}{8}}=-\frac{R}{14}$
$\therefore$ shift $=\frac{R}{14}$
(d)

Angular momentum is given by

$$
\begin{aligned}
J= & I \omega=\left(\frac{2 M R^{2}}{5}\right) \omega \\
& =\frac{2 M R^{2}}{5} \times \frac{2 \pi}{T}=\frac{4 \pi M R^{2}}{5 T}
\end{aligned}
$$

(a)

Acceleration of each mass $=a=\left(\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right) g$
Now acceleration of centre of mass of the system

$$
A_{c m}=\frac{m_{1} \overrightarrow{a_{1}}+m_{1} \overrightarrow{a_{2}}}{m_{1}+m_{2}}
$$

As both masses move with same acceleration but in opposite direction so $\overrightarrow{a_{1}}=-\overrightarrow{a_{2}}=a$ (let)
$a \downarrow\left[\frac{m_{1}}{m_{2}} a \uparrow\right.$
$\therefore A_{c m}=\frac{m_{1} a-m_{2} a}{m_{1}+m_{2}}$
$=\left(\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right) \times\left(\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right) \times g=\left(\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right)^{2} \times g$
(c)

If speed of man relative to plank be $v$, then it can be shown easily that speed of man relative to ground
$v_{\mathrm{mg}}=v \frac{M}{\left(M+\frac{M}{3}\right)}=\frac{3}{4} v$
$\therefore$ Distance covered by man relative to ground
$=L \frac{v_{\mathrm{mg}}}{v}=\frac{L}{v} \frac{3}{4} v=\frac{3 L}{4}$
(b)
M.I. of disc $=\frac{1}{2} M R^{2}=\frac{1}{2} M\left(\frac{M}{\pi t \rho}\right)=\frac{1 M^{2}}{2 \pi t \rho}$
(As $\rho=\frac{M}{\pi R^{2} t}$ There fore $\left.\mathrm{R}^{2}=\frac{M}{\pi t \rho}\right)$
If mass and thickness are same then, $I \propto \frac{1}{\rho}$
$\therefore \frac{I_{1}}{I_{2}}=\frac{\rho_{2}}{\rho_{1}}=\frac{3}{1}$
(a)

If $M=M^{\prime}$ then bullet will transfer whole of its velocity (and consequently $100 \%$ of its KE) to block and will itself come to rest as per theory of collision.
(c)

Acceleration $a=\frac{v_{-} u}{t}$
Or $a=\frac{v-v_{0}}{t}$

Or $\mathrm{g}=\frac{v_{-} v_{0}}{t}$
$\therefore \quad v=0$
Speed before first bounce
$v_{0}=-5 \mathrm{~ms}^{-1}$
$\therefore t=\frac{v_{B}-v_{A}}{\mathrm{~g}}=\frac{0(-5)}{10}=\frac{5}{10}=0.5 \mathrm{~s}$
(a)
$m=0.6 \mathrm{~kg}$


Mass per unit length $=\frac{0.6}{100} \mathrm{kgcm}^{-1}$
Mass of part $A B, m_{1}=\frac{0.6}{100} \times 20=\frac{0.6}{5} \mathrm{~kg}$
Mass of part $B C, m_{2}=\frac{0.6}{100} \times 80$
Moment of inertia $=\frac{0.6 \times 4}{5}=\frac{2.4}{5} \mathrm{~kg}$

$$
\begin{aligned}
I & =m_{1}\left(\frac{A B}{2}\right)^{2}+m_{2}\left(\frac{B C}{2}\right)^{2} \\
& =\frac{0.6}{5} \times\left(\frac{20}{2} \times 10^{-2}\right)^{2}+\frac{24}{5} \times\left(\frac{80}{2} \times 10^{-2}\right)^{2} \\
& =\frac{0.6}{5} \times 10^{-2}+\frac{2.4}{5} \times\left(4 \times 10^{-1}\right)^{2} \\
& =\frac{0.6}{5} \times 10^{-2}+\frac{2.4}{5} \times 16 \times 10^{-2} \\
& =\left(\frac{0.6+38.4}{5}\right) \times 10^{-2} \\
& =7.8 \times 10^{-2} \mathrm{~kg}-\mathrm{m}^{2}=0.078 \mathrm{~kg}-\mathrm{m}^{2}
\end{aligned}
$$

(a)
$P=\sqrt{p_{x}^{2}+p_{y}^{2}}$
$=\sqrt{(2 \cos t)^{2}+(2 \sin t)^{2}}=2$
If $m$ be the mass of the body, then kinetic energy
$=\frac{p^{2}}{2 m}=\frac{(2)^{2}}{2 m}=\frac{2}{m}$
Since kinetic energy does not change with time, both work done and power are zero
Now Power $=F v \cos \theta=0$
As $F \neq 0, v \neq 0$
$\therefore \cos \theta=0$
Or $\theta=90^{\circ}$
As direction of $\overrightarrow{\mathrm{p}}$ is same that $\overrightarrow{\mathrm{v}}(\because \overrightarrow{\mathrm{p}}=m \overrightarrow{\mathrm{v}})$ hence angle between $\overrightarrow{\mathrm{F}}$ and $\overrightarrow{\mathrm{p}}$ is equal to $90^{\circ}$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| A. | C | C | C | C | D | B | D | D | C | B |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
| A. | D | A | C | B | A | D | C | B | A | A |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |



