CLASS : XITh
Solutions

## Topic:- oscillations

2

3

4

5

6

7

8
(a)

Potential energy is minimum (in the case zero) at mean position ( $x=0$ ) and maximum at extreme positions $(x= \pm A)$. At time $t=0, x=A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x$ $=0$. Hence, this is also correct.
(b)
$v_{\text {max }}=a \omega$ and
Maximum acceleration $=\omega^{2} a$

$$
=\left(\frac{v}{a}\right)^{2} a=\frac{v^{2}}{a}
$$

(b)

When bigger pendulum of time period ( $5 T / 4$ ) completes one vibration, the smaller pendulum will complete ( $5 / 4$ ) vibrations. It means the smaller pendulum will be leading the bigger pendulum by phase $T / 4 \mathrm{sec}=\pi / 2 \mathrm{rad}=90^{\circ}$
(b)

Time period of simple pendulum

$$
T=2 \pi \sqrt{\frac{l}{g}}
$$

From this formula it can be predicted that time period does not depend on the mass of bob.
(d)
$T \propto \sqrt{l} \Rightarrow \frac{T_{1}}{T_{2}}=\sqrt{\frac{l_{1}}{l_{2}}} \Rightarrow \frac{2}{T_{2}}=\sqrt{\frac{l}{4 l}} \Rightarrow T_{2}=4 \mathrm{~s}$
(c)
$y=a \sin \frac{2 \pi}{T} t \Rightarrow \frac{a}{2}=a \sin \frac{2 \pi t}{3} \Rightarrow \frac{1}{2}=\sin \frac{2 \pi t}{3}$ $\Rightarrow \sin \frac{2 \pi t}{3}=\sin \frac{\pi}{6} \Rightarrow \frac{2 \pi t}{3}=\frac{\pi}{6} \Rightarrow t=\frac{1}{4} \sec$
(b)
$T=2 \pi \sqrt{\frac{m}{k}}$ or $k=\frac{4 \pi^{2}}{T^{2}} m=\omega^{2} m$
(b)

So $a=6 \mathrm{~cm}, \omega=100 \mathrm{rad} / \mathrm{s}$
$K_{\text {max }}=\frac{1}{2} m \omega^{2} a^{2}=\frac{1}{2} \times 1 \times(100)^{2} \times\left(6 \times 10^{-2}\right)^{2}=18 \mathrm{~J}$
(d)

As retardation $=b v$
$\therefore$ retarding force $=m b v$
$\therefore$ net restoring torque when angular displacement is $\theta$ is given by

$=-m g \ell \sin \theta+m b v \ell$
$\therefore I \alpha=-m g \ell \sin \theta+m b v \ell$
where, $I=m \ell^{2}$
$\therefore \frac{d^{2} \theta}{d t^{2}}=\alpha=-\frac{g}{\ell} \sin \theta+\frac{b v}{\ell}$
for small damping, the solution of the above differential equation will be
$\therefore \theta=\theta_{0} e^{-\frac{b t}{2}} \sin (\omega t+\phi)$
$\therefore$ angular amplitude will be $=\theta . e^{\frac{b t}{2}}$
According to question, in $\tau$ time (average life-time),
Angular amplitude drops to $\frac{1}{e}$ value of its original value $(\theta)$
$\therefore \frac{\theta_{0}}{e}=\theta_{0} e^{-\frac{b \tau}{2}} \Rightarrow \frac{b \tau}{2}=1$
$\therefore \tau=\frac{2}{b}$
11 (c)
Effective value of acceleration due to gravity is zero in the satellite, $i e, \mathrm{~g}_{\text {eff }}=0$.
Hence, time period of pendulum

$$
T=2 \pi \sqrt{\frac{l}{g_{\text {eff }}}}=2 \pi \sqrt{\frac{l}{0}}=\infty
$$

is infinite.
(d)
$T \propto \frac{1}{\sqrt{k}} \Rightarrow \frac{T_{2}}{T_{1}}=\sqrt{\frac{k_{1}}{k_{2}}}=\sqrt{\frac{k}{4 k}}=\frac{1}{2} \Rightarrow T_{2}=\frac{T_{1}}{2}$
(c)

The relation between acceleration ( $a$ ) and displacement $(x)$ for a body in SHM is

$$
a=-\omega^{2} x
$$

Given, $\quad a=-b x$
On comparing the two equations, we get

$$
\begin{array}{llrl} 
& & \omega^{2} & =b \\
& \therefore & & \omega=\sqrt{b} \\
\text { Since, } & & \omega=\frac{2 \pi}{T} \\
& \therefore & & \frac{2 \pi}{T}=\sqrt{b} \\
\Rightarrow & & T & =\frac{2 \pi}{\sqrt{b}}
\end{array}
$$

(d)

The lift is moving with constant velocity so, there will be no change in the acceleration hence time period will remain same.
(a)

Here, Mass of the body, $m=500 \mathrm{~g}=500 \times 10^{-3} \mathrm{~kg}$
Spring constant, $k=8 \pi^{2} \mathrm{Nm}^{-1}$
The frequency of oscillation is
$v=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}=\frac{1}{2 \pi} \sqrt{\frac{8 \pi^{2} \mathrm{~N} \mathrm{~m}^{-1}}{500 \times 10^{-3} \mathrm{~kg}}}=2 \mathrm{~Hz}$
(c)

Kinetic energy at mean position,
$K_{\max }=\frac{1}{2} m v_{\text {max }}^{2} . \Rightarrow v_{\max }=\sqrt{\frac{2 K_{\max }}{m}}$
$=\sqrt{\frac{2 \times 16}{0.32}}=\sqrt{100}=10 \mathrm{~m} / \mathrm{s}$
(c)

In SHM, the total energy=potential energy + kinetic energy
or $\quad E=U+K$

$$
\begin{aligned}
& =\frac{1}{2} m \omega^{2} x^{2}+\frac{1}{2} m \omega^{2}\left(A^{2}-x^{2}\right) \\
& =\frac{1}{2} m \omega^{2} A^{2}=\frac{1}{2} k A^{2}
\end{aligned}
$$

where $k=$ force constant $=m \omega^{2}$
Thus, total energy depends on $k$ and $A$.
(a)
$n \propto \sqrt{\frac{k}{m}}$
(d)
$F_{\text {max }}=m \omega^{2} a=m \frac{4 \pi^{2}}{T^{2}} a$

$$
=\frac{10}{1000} \times \frac{4 \times(\pi)^{2}}{(\pi / 5)} \times 0.5=0.5 \mathrm{~N}
$$

(a)

Max, $\mathrm{KE}=$ Max. PE
$=\frac{1}{2} m v^{2}=\frac{1}{2} k x^{2}=\frac{1}{2} \times 65 \times(0.11)^{2}$
or $v^{2}=\frac{65 \times(0.11)^{2}}{650 \times 10^{-3}}$ or $v=1.1 \mathrm{~ms}^{-1}$


| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| A. | A | A | B | B | B | D | C | B | B | D |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
| A. | C | D | C | D | A | C | C | A | D | A |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |



