

When a mass *m* is placed on mass*M*, the new system is of mass = (M + m), attached to the spring. New time period of oscillation,

$$T' = 2\pi \sqrt{\frac{M+m}{k}}$$
$$T = 2\pi \sqrt{\frac{M}{k}}$$

Let v = velocity of the mass M while passing through the mean position. v' = Velocity of the mass(M + m), while passing through the mean position. According to law of conservation of linear momentum Mv = (M + m)v'At mean position, $v = A \omega$ and $v' = A'\omega'$

$$\therefore MA\omega = (m+m)A'\omega$$

or $A' = \left(\frac{M}{M+m}\right)\frac{\omega}{\omega'}A = \frac{M}{M+m} \times \frac{T'}{T} \times A$
$$= \left(\frac{M}{M+m}\right) \times \sqrt{\frac{M+m}{M}} \times A$$
$$= A\sqrt{\frac{M}{M+m}}$$

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(c) $T = 2\pi \sqrt{\frac{M}{k}}$ when mass is increased by *m* then ...(i) $T = 2\pi \sqrt{\frac{M+m}{k}}$ $\frac{5T}{3} = 2\pi \sqrt{\frac{M+m}{\nu}}$...(ii) ⇒ Dividing Eq. (i) by Eq<mark>. (ii), we get</mark> $\frac{3}{5} = \sqrt{\frac{M}{M+m}}$ $\frac{9}{25} = \frac{M}{M+m}$ 9M + 9m = 25M⇒ 16M = 9m⇒ $\frac{m}{M} = \frac{16}{9}$ (a) $T \propto \frac{1}{\sqrt{k}} \Rightarrow T_1: T_2: T_3 = \frac{1}{\sqrt{k}}: \frac{1}{\sqrt{k/2}}: \frac{1}{\sqrt{2k}} = 1:\sqrt{2}: \frac{1}{\sqrt{2}}$ (a) $y_1 = 4\sin\left(4\pi t + \frac{\pi}{2}\right) = 4\cos 4\pi t$ $y_2 = 3\cos(4\pi t) = 3\cos 4\pi t$ The phase difference = 0, both are along the same line $\therefore A^2 = 4^2 + 3^2 + 2 \times 4 \times 3 \cos 0^\circ$ $A^2 = (4+3)^2 \Rightarrow A = 7$ The resultant amplitude is 7 units **(b)**

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$$n = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \Rightarrow n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{L_2}{2L_2}}$$
$$\Rightarrow \frac{n_1}{n_2} = \frac{1}{\sqrt{2}} \Rightarrow n_2 = \sqrt{2}n_1 \Rightarrow n_2 > n_1$$
Energy $E = \frac{1}{2}m\omega^2 a^2 = 2\pi^2 m n^2 a^2$
$$\Rightarrow \frac{a_1^2}{a_2^2} = \frac{m_2 n_2^2}{m_1 n_1^2} \qquad [\because E \text{ is same}]$$
Given $n_2 > n_1$ and $m_1 = m_2 \Rightarrow a_1 > a_2$

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(b)

(a)

(c)

(a)

The two spring on left side having spring constant of 2k each are in series, equivalent constant is $\frac{1}{\left(\frac{1}{2k} + \frac{1}{2k}\right)} = k$. The two springs on right hand side of mass M are in parallel. Their effective spring constant is (k + 2k) = 3k

Equivalent spring constants of value k and 3k are in parallel and their net value of spring constant of all the four springs is k + 3k = 4k

$$\therefore \text{ Frequency of mass is } n = \frac{1}{2\pi} \sqrt{\frac{4k}{M}}$$

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For S.H.M. F = -kx

 \therefore Force = Mass \times Acceleration $\propto -x$

 \Rightarrow *F* = -Akx; where *A* and *k* are positive constants

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When lift accelerates upwards, then effective acceleration on the pendulum

$$g_{eff} = g + \frac{g}{3} = \frac{4g}{3}$$

∴ Time period $T' = 2\pi \sqrt{\frac{l}{g_{eff}}} = 2\pi \sqrt{\frac{l}{4g/3}}$

$$= \frac{\sqrt{3}}{2} \cdot 2\pi \sqrt{\frac{l}{g}}$$

$$= \frac{\sqrt{3}}{2} T$$

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When particle is at x = 2, the displacement is $y = 4 \sim 2 = 2$ cm. If r is the time taken by the particle to go from x = 4 cm to x = 2 cm, then

$$y = a \cos \omega t = a \cos \frac{2\pi t}{T} = a \cos \frac{2\pi t}{1.2}$$

or $\cos \frac{2\pi t}{1.2} = \frac{y}{a} = \frac{2}{4} = \frac{1}{2} = \cos \frac{\pi}{3}$
or $\frac{2t}{1.2} = \frac{1}{3}$ or $t = \frac{1.2}{6} = 0.26$
time taken to move from $x = +2$ cm to $x = +4$ cm and back again
 $= 2t = 2 \times 0.2$ s $= 0.4$ s
(a)

Under forced oscillations, the body will vibrate with the frequency of the driving force

14 **(c)**

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$$E = \frac{1}{2}m\omega^2 a^2 \Rightarrow \frac{E'}{E} = \frac{a'^2}{a^2} \Rightarrow \frac{E'}{E} = \frac{\left(\frac{3}{4}a\right)^2}{a^2} \left(\because a' = \frac{3}{4}a\right)$$
$$\Rightarrow E' = \frac{9}{16}E$$
(b)

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Let at any instant, cube is at a depth x from the equilibrium position then net force acting on the cube = upthrust on the portion of length x

$$F = -\rho l^2 x g = -\rho l^2 g x \qquad \dots (i)$$

Negative sign shows that, force is opposite to *x*. Hence equation of SHM

$$= -kx$$

Comparing Eqs. (i) and (ii)

$$k = \rho l^2 g$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi \sqrt{\frac{l d}{\rho g}}$$

F

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(a)

We know that during SHM, the restoring force is proportional to the displacement from equilibrium position. Hence restoring force is maximum when the displacement is maximum at its extreme position

17 **(b)**

Kinetic energy in SHM

$$KE = \frac{1}{2}m\omega^{2}(A^{2} - x^{2})$$

At $x = \frac{A}{2}$
$$KE = \frac{1}{2}m\omega^{2}(A^{2} - \frac{A^{2}}{4}) = \frac{3}{4}(\frac{1}{2}m\omega^{2}A^{2})$$
$$= \frac{3}{4} \times \text{ Total energy of the particle}$$

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(c)

Displacement equation

 $y = A\sin \omega t \, B \cos \omega t$ Let $A = a\cos \theta$ and $B = a\sin \theta$ So, $A^2 + B^2 = a^2$ $\Rightarrow \quad a = \sqrt{A^2 + B^2}$

Then, $y = a\cos\theta\sin\omega t - a\sin\theta\cos\omega t$

 $y = a \sin(\omega t \cdot \theta)$

which is the equation of simple harmonic oscillator The amplitude of the oscillator

$$=a=\sqrt{A^2+B^2}$$

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(d)

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{\frac{l}{g}}$$
, it is does not depend upon mass

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(c) When t = 1 s, $y_1 = r\sin \omega \times 1 = r\sin \omega$ When t = 2 s, $y_2 = r\sin \omega \times 2 = r\sin 2\omega$

$$\therefore \frac{y_1}{y_2} = \frac{r \sin \omega}{r \sin 2\omega}$$

$$= \frac{1}{2 \cos \omega} = \frac{1}{2 \cos 2\pi/T}$$

$$= \frac{1}{2 \cos 2\pi/8}$$

$$= \frac{1}{2 (1/\sqrt{2})} = \frac{1}{\sqrt{2}}$$

$$\therefore y_2 = \sqrt{2}y_1$$
Distance converted in 2nd second
$$= y_2 - y_1 = (\sqrt{2} - 1)y_1$$

$$\therefore \text{ Ratio} = 1:(\sqrt{2} - 1)$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	С	С	С	С	С	A	A	В	В	А
Q.	11	12	13	14	15	16	17	18	19	20
A.	С	А	A	С	В	A	В	С	D	С

