CLASS : XITh

## Solutions

## Topic:- osCillations

1
(c)

When the bob falls through a vertical height of 1 m , the velocity acquired at the lowest point,
$v=\sqrt{2 g \mathrm{~h}}=\sqrt{2 \times 10 \times 1}=\sqrt{20} \mathrm{~ms}^{-1}$
Centrifugal force $=\frac{m v^{2}}{r}=\frac{0.01 \times 20}{1}=0.20 \mathrm{~N}$
Net tension=weight + centrifugal force

$$
=(0.01 \times 10+0.20)=0.30 \mathrm{~N}
$$

(c)

Mass $(m)=20 \mathrm{~g}=0.02 \mathrm{~kg}$
Frequency $(f)=\frac{5}{\pi} \mathrm{~Hz}$
Time period of a loaded spring

$$
T=2 \pi \sqrt{\frac{m}{k}}
$$

Frequency $(f)=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}$

$\frac{5}{\pi}=\frac{1}{2 \pi} \sqrt{\frac{k}{0.02}}$
or

$$
10=\sqrt{\frac{k}{0.02}}
$$

or $\quad 100=\frac{k}{0.02}$
$\therefore \quad k=2 \mathrm{Nm}^{-1}$
(c)
$y_{1}=a \sin \left(\omega t_{-} k x\right)$;
$y_{2}=b \cos \left(\omega t-\frac{k}{x}\right)=b \sin \left(\omega t-\frac{k}{x}+\pi / 2\right)$
$\therefore$ Phase difference $=\left(\omega t-\frac{k}{x}+\pi / 2\right)-(\omega t-k x)$
$=\pi / 2$
(c)

When a mass $m$ is placed on mass $M$, the new system is of mass $=(M+m)$, attached to the spring. New time period of oscillation,
$\begin{aligned} T^{\prime} & =2 \pi \sqrt{\frac{M+m}{k}} \\ T & =2 \pi \sqrt{\frac{M}{k}}\end{aligned}$
Let $v=$ velocity of the mass $M$ while passing through the mean position. $v^{\prime}=$ Velocity of the mass $(M+m)$, while passing through the mean position.
According to law of conservation of linear momentum $M v=(M+m) v^{\prime}$
At mean position, $v=A \omega$ and $v^{\prime}=A^{\prime} \omega^{\prime}$
$\therefore \quad M A \omega=(m+m) A^{\prime} \omega$
or $A^{\prime}=\left(\frac{M}{M+m} \frac{\omega}{\omega}, A=\frac{M}{M+m} \times \frac{T^{\prime}}{T} \times A\right.$
$=\left(\frac{M}{M+m}\right) \times \sqrt{\frac{M+m}{M}} \times A$
$=A \sqrt{\frac{M}{M+m}}$
$n=\frac{1}{2 \pi} \sqrt{\frac{g}{l}} \Rightarrow n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_{1}}{n_{2}}=\sqrt{\frac{l_{2}}{l_{1}}}=\sqrt{\frac{L_{2}}{2 L_{2}}}$
$\Rightarrow \frac{n_{1}}{n_{2}}=\frac{1}{\sqrt{2}} \Rightarrow n_{2}=\sqrt{2} n_{1} \Rightarrow n_{2}>n_{1}$
Energy $E=\frac{1}{2} m \omega^{2} a^{2}=2 \pi^{2} m n^{2} a^{2}$
$\Rightarrow \frac{a_{1}^{2}}{a_{2}^{2}}=\frac{m_{2} n_{2}^{2}}{m_{1} n_{1}^{2}} \quad[\because E$ is same $]$
Given $n_{2}>n_{1}$ and $m_{1}=m_{2} \Rightarrow a_{1}>a_{2}$
(b)

The two spring on left side having spring constant of $2 k$ each are in series, equivalent constant is $\frac{1}{\left(\frac{1}{2 k}+\frac{1}{2 k}\right)}=k$. The two springs on right hand side of mass $M$ are in parallel. Their effective spring constant is $(k+2 k)=3 k$
Equivalent spring constants of value $k$ and $3 k$ are in parallel and their net value of spring constant of all the four springs is $k+3 k=4 k$
$\therefore$ Frequency of mass is $n=\frac{1}{2 \pi} \sqrt{\frac{4 k}{M}}$
(a)

For S.H.M. $F=-k x$
$\therefore$ Force $=$ Mass $\times$ Acceleration $\propto-x$
$\Rightarrow F=-A k x$; where $A$ and $k$ are positive constants
(c)

When lift accelerates upwards, then effective acceleration on the pendulum

$$
\begin{aligned}
\mathrm{g}_{\text {eff }}=\mathrm{g} & +\frac{\mathrm{g}}{3}=\frac{4 \mathrm{~g}}{3} \\
\therefore \text { Time period } T^{\prime} & =2 \pi \sqrt{\frac{l}{\text { geff }}}=2 \pi \sqrt{\frac{l}{4 \mathrm{~g} / 3}} \\
& =\frac{\sqrt{3}}{2} \cdot 2 \pi \sqrt{\frac{l}{\mathrm{~g}}} \\
& =\frac{\sqrt{3}}{2} T
\end{aligned}
$$

(a)

When particle is at $x=2$, the displacement is $y=4 \sim 2=2 \mathrm{~cm}$. If $r$ is the time taken by the particle to go from $x=4 \mathrm{~cm}$ to $x=2 \mathrm{~cm}$, then
$y=a \cos \omega t=a \cos \frac{2 \pi t}{T}=a \cos \frac{2 \pi t}{1.2}$
or $\cos \frac{2 \pi t}{1.2}=\frac{y}{a}=\frac{2}{4}=\frac{1}{2}=\cos \frac{\pi}{3}$
or $\frac{2 t}{1.2}=\frac{1}{3}$ or $t=\frac{1.2}{6}=0.26$
time taken to move from $x=+2 \mathrm{~cm}$ to $x=+4 \mathrm{~cm}$ and back again
$=2 t=2 \times 0.2 \mathrm{~s}=0.4 \mathrm{~s}$
(a)

Under forced oscillations, the body will vibrate with the frequency of the driving force
(c)
$E=\frac{1}{2} m \omega^{2} a^{2} \Rightarrow \frac{E^{\prime}}{E}=\frac{a^{\prime 2}}{a^{2}} \Rightarrow \frac{E^{\prime}}{E}=\frac{\left(\frac{3}{4} a\right)^{2}}{a^{2}}\left(\because a^{\prime}=\frac{3}{4} a\right)$
$\Rightarrow E^{\prime}=\frac{9}{16} E$
(b)

Let at any instant, cube is at a depth $x$ from the equilibrium position then net force acting on the cube $=$ upthrust on the portion of length $x$

$$
\begin{equation*}
F=-\rho l^{2} x \mathrm{~g}=-\rho l^{2} \mathrm{~g} x \tag{i}
\end{equation*}
$$

Negative sign shows that, force is opposite to $x$. Hence equation of SHM

$$
\begin{equation*}
F=-k x \tag{ii}
\end{equation*}
$$

Comparing Eqs. (i) and (ii)

$k=\rho l^{2} \mathrm{~g}$
$T=2 \pi \sqrt{\frac{m}{k}}$

$$
=2 \pi \sqrt{\frac{l^{3} d}{\rho l^{2} \mathrm{~g}}}=2 \pi \sqrt{\frac{l d}{\rho \mathrm{~g}}}
$$

(a)

We know that during SHM, the restoring force is proportional to the displacement from equilibrium position. Hence restoring force is maximum when the displacement is maximum at its extreme position

## (b)

Kinetic energy in SHM

$$
\begin{aligned}
& \mathrm{KE}=\frac{1}{2} m \omega^{2}\left(A^{2}-x^{2}\right) \\
& \text { At } x=\frac{A}{2} \\
& \begin{aligned}
\mathrm{KE} & =\frac{1}{2} m \omega^{2}\left(A^{2}-\frac{A^{2}}{4}\right)=\frac{3}{4}\left(\frac{1}{2} m \omega^{2} A^{2}\right) \\
& =\frac{3}{4} \times \text { Total energy of the particle }
\end{aligned}
\end{aligned}
$$

(c)

Displacement equation

$$
y=A \sin \omega t_{-} B \cos \omega t
$$

Let $A=a \cos \theta \quad$ and $\quad B=a \sin \theta$
So, $\quad A^{2}+B^{2}=a^{2}$
$\Rightarrow \quad a=\sqrt{A^{2}+B^{2}}$
Then, $\quad y=a \cos \theta \sin \omega t-a \sin \theta \cos \omega t$

$$
y=a \sin \left(\omega t_{-} \theta\right)
$$

which is the equation of simple harmonic oscillator The amplitude of the oscillator

$$
=a=\sqrt{A^{2}+B^{2}}
$$

(d)
$T=2 \pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{\frac{l}{g}}$, it is does not depend upon mass
(c)

When $t=1 s, y_{1}=r \sin \omega \times 1=r \sin \omega$
When $t=2 s, y_{2}=r \sin \omega \times 2=r \sin 2 \omega$
$\therefore \frac{y_{1}}{y_{2}}=\frac{r \sin \omega}{r \sin 2 \omega}$
$=\frac{1}{2 \cos \omega}=\frac{1}{2 \cos 2 \pi / T}$
$=\frac{1}{2 \cos 2 \pi / 8}$
$=\frac{1}{2 \cos \pi / 4}$
$=\frac{1}{2(1 / \sqrt{2})}=\frac{1}{\sqrt{2}}$
$\therefore \quad y_{2}=\sqrt{2} y_{1}$
Distance converted in $2^{\text {nd }}$ second
$=y_{2}-y_{1}=(\sqrt{2}-1) y_{1}$
$\therefore \quad$ Ratio $=1:(\sqrt{2}-1)$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |
| A. | C | C | C | C | C | A | A | B | B | A |  |  |  |
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| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |  |
| A. | C | A | A | C | B | A | B | C | D | C |  |  |  |
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