

## Topic :- OSCILLATIONS

- 1 (c)  
Under the influence of one force  $F_1 = m\omega_1^2 y$  and under the action of another force,  $F_2 = m\omega_2^2 y$

Under the action of both the forces  $F = F_1 + F_2$

$$\Rightarrow m\omega^2 y = m\omega_1^2 y + m\omega_2^2 y$$

$$\Rightarrow \omega^2 = \omega_1^2 + \omega_2^2 \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$$

$$\Rightarrow T = \frac{\sqrt{T_1^2 T_2^2}}{\sqrt{T_1^2 + T_2^2}} = \frac{\left(\frac{4}{5}\right)^2 \left(\frac{3}{5}\right)^2}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = 0.48 \text{ s}$$

- 2 (b)  
 $T = 2\pi \sqrt{\frac{m}{k}}, T' = 2\pi \sqrt{\frac{m}{4k}} = \frac{T}{2} = \frac{5}{2} \text{ s} = 2.5 \text{ s}$

- 3 (c)  
If  $v$  and  $v'$  are the velocities of the block of mass  $M$  and  $(M + m)$  while passing from the mean position when executing SHM

Using law of conservation of linear momentum, we have

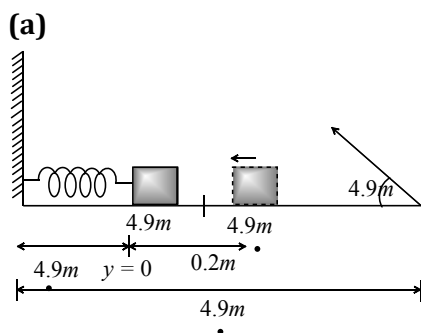
$$mv = (M + m)v' \text{ or } v' = mv/(M + m)$$

Also, maximum PE = maximum KE

$$\therefore \frac{1}{2} k A^2 = \frac{1}{2} (M + m)v'^2$$

$$\text{or } A' = \left(\frac{M + m}{k}\right)^{1/2} \times \frac{mv}{(M + m)}$$
$$= \frac{mv}{\sqrt{(M + m)k}}$$

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The block is released from A

$$x = 4.9m + (0.2m) \sin\left(\omega t + \frac{\pi}{2}\right)$$

at  $t = 1s; x = 5m$

so range of projectile will be 5m

$$\text{Now } 5 = \frac{v^2 \sin 90^\circ}{g} \Rightarrow v^2 = 50 \Rightarrow v = \sqrt{50}$$

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**(c)**

On comparing with standard equation  $\frac{d^2y}{dt^2} + \omega^2 y = 0$

$$\text{we get } \omega^2 = K \Rightarrow \omega = \frac{2\pi}{T} = \sqrt{K} \Rightarrow T = \frac{2\pi}{\sqrt{K}}$$

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**(c)**

$$y = A \sin\left(\frac{2\pi}{T}t\right)$$

$$\frac{A}{2} = A \sin\left(\frac{2\pi}{T}t\right) = \frac{2\pi}{T}t = \pi/6$$

$$\text{Time period } t = \frac{T}{12} = \frac{6}{12} = \frac{1}{2} \text{ s}$$

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**(d)**

Potential energy of particle performing SHM is given by:  $PE = \frac{1}{2}m\omega^2 y^2$ , i.e., it varies parabolically such that at mean position it becomes zero and maximum at extreme positions

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**(c)**

A particle oscillating under a force  $\vec{F} = -k\vec{x} = b\vec{v}$  is a damped oscillator. The first term  $-k\vec{x}$  represents the restoring force and second term  $-b\vec{v}$  represents the damping force

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**(c)**

Effective force constant is equal to the reciprocal of the sum of individual force constant, hence

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots$$

Given,  $k_1 = k, k_2 = 2k, k_3 = 3k, \dots$

$$\therefore \frac{1}{k_e} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \dots$$

The given series a geometric progression series, hence sum is

$$S_\infty = \frac{a}{1-r}$$

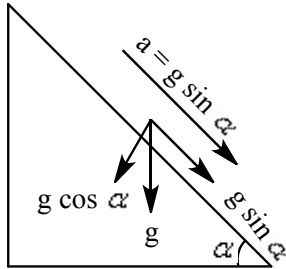
where  $a$  is first term of series and  $r$  the common difference.

$$\Rightarrow \frac{1}{k_e} = \frac{1}{k} \times \frac{1}{\left(1 - \frac{1}{2}\right)} = \frac{2}{k} \Rightarrow k_e = \frac{k}{2}$$

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(a)

Time period  $T = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}}$



$$T = 2\pi \sqrt{\frac{l}{g \cos \alpha}}$$

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(d)

$$x_1 + x_2 = A \text{ and } k_1 x_1 = k_2 x_2 \text{ or } \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

Solving these equations, we get

$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)A$$

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(c)

The effective acceleration in a lift descending with acceleration  $\frac{g}{3}$  is  $g_{\text{eff}} = g - \frac{g}{3} = \frac{2g}{3}$

$$\therefore T = 2\pi \sqrt{\left(\frac{L}{g_{\text{eff}}}\right)} = 2\pi \sqrt{\left(\frac{L}{2g/3}\right)} = 2\pi \sqrt{\left(\frac{3L}{2g}\right)}$$

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(b)

Time period of simple pendulum is given by

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \dots(i)$$

When the lift is moving up with an acceleration  $a$ , then time period becomes

$$T' = 2\pi \sqrt{\frac{l}{g+a}}$$

Here,  $T' = \frac{T}{2}$

$$\Rightarrow \frac{T}{2} = 2\pi \sqrt{\frac{l}{g+a}} \quad \dots(ii)$$

Dividing Eq.(ii) by Eq. (i), we get

$$a = 3g$$

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(d)

$$n = \frac{1}{2\pi} \sqrt{\frac{K_{eq}}{m}} = \frac{1}{2\pi} \sqrt{\frac{K_1 K_2}{(K_1 + K_2)m}}$$

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**(b)**

$m_1 = 1 \text{ kg}$ , extension  $l_1 = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

$$\therefore m_1 g = k l_1$$

$k =$  force constant of the spring

$$k = \frac{m_1 g}{l_1} = \frac{1 \times 10}{5 \times 10^{-2}} = 200 \text{ Nm}^{-1}$$

Time period of the block of mass 2 kg.

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{2}{200}} = 2\pi \times \frac{1}{10} = \frac{\pi}{5} \text{ s}$$

Maximum velocity  $v_{\max} = A\omega$

where  $A =$  Amplitude

$$= 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$$

$$v_{\max} = A \times \frac{2\pi}{T} = 10 \times 10^{-2} \times \frac{2\pi}{\pi/5}$$

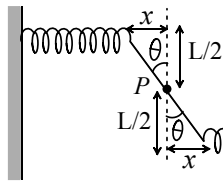
$$= 10^{-1} \times 2 \times 5 = 1 \text{ ms}^{-1}$$

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**(c)**

Torque about  $P = -(kx)\frac{L}{2} + (-kx\frac{L}{2}) = -kxL = -k\frac{L^2}{2}\theta$

For small angle  $\theta$ ,  $x = \frac{L}{2}\theta$ ;  $\tau = -I\alpha$



$$\Rightarrow -\frac{KL^2}{2}\theta = \frac{ML^2}{12}\alpha$$

$$\Rightarrow \frac{-6K\theta}{M} = \alpha$$

$$\Rightarrow \omega = \sqrt{\frac{6K}{M}} \text{ and } f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{6K}{M}}$$

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**(c)**

If first equation is

$$y_1 = a_1 \sin \omega t \Rightarrow \sin \omega t = \frac{y_1}{a_1} \quad \dots(i)$$

Then second equation will be

$$y_2 = a_2 \sin(\omega t + \frac{\pi}{2})$$

$$= a_2 [\sin \omega t \cos \frac{\pi}{2} + \cos \omega t \sin \frac{\pi}{2}] = a_2 \cos \omega t$$

$$\Rightarrow \cos \omega t = \frac{y_2}{a_2} \quad \dots(ii)$$

By squaring and adding Eqs. (i) and (ii)

$$\sin^2 \omega t + \cos^2 \omega t = \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2}$$

$$\Rightarrow \frac{y_1^2}{a_1^2} + \frac{y_2^2}{a_2^2} = 1;$$

This is equation of an ellipse.

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**(c)**

By using conservation of mechanical energy

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow x = v\sqrt{m/k}$$

PE

<b>ANSWER-KEY</b>										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	A	C	C	D	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	C	B	B	D	B	C	C	C

PE