CLASS : XITh
Solutions

## Topic:- oscililations

1

2
as
For
(b)
$T \propto \sqrt{l}, \because$ effective length $l_{\text {sitting }}>l_{\text {standing }}$
(a)

Let the equations of two mutually perpendicular SHM's same frequency be

$$
x=a_{1} \sin \omega t \text { and } y=a_{2} \sin (\omega t+\emptyset)
$$

Then, the general equation of Lissajous figure can be obtained

$$
\begin{aligned}
& \text { as } \quad \frac{x^{2}}{a_{1}^{2}}+\frac{y^{2}}{a_{2}^{2}}-\frac{2 x y}{a_{1} a_{2}} \cos \emptyset=\sin ^{2} \emptyset \\
& \text { For } \quad \emptyset=0^{\circ}:: \frac{x^{2}}{a_{1}^{2}}+\frac{y^{2}}{a_{2}^{2}}-\frac{2 x y}{a_{1} a_{2}}=0 \\
& \Rightarrow\left[\frac{x}{a_{1}}-\frac{y}{a_{2}}\right]^{2}=0 \Rightarrow \frac{x}{a_{1}}=\frac{y}{a_{2}} \Rightarrow y=\frac{a_{2}}{a_{1}} x
\end{aligned}
$$

This is an equation of a straight line passing through origin.
(a)

In this case springs are in parallel, so $k_{e q}=k_{1}+k_{2}$
and $\omega=\sqrt{\frac{k_{e q}}{m}}=\sqrt{\frac{k_{1}+k_{2}}{m}}$
(c)
$x=A \sin (\omega+\pi / 2)=A \cos \omega t$
$\therefore \cos \omega t=x / A$ and $\sin \omega t=\sqrt{1-\left(x^{2} / A^{2}\right)}$
$y=A \sin 2 \omega t=\sqrt{1-\left(x^{2} / \Delta A^{2}\right)}$
$y=A \sin 2 \omega t=2 A \sin \omega t \cos \omega t$
or $y^{2}=4 A^{2} \sin ^{2} \omega \mathrm{t} \cos ^{2} \omega \mathrm{t}$
$=4 A^{2} \times \frac{x^{2}}{A^{2}} \times\left(\frac{A^{2}-x^{2}}{A^{2}}\right)=4 x^{2}\left(1-\frac{x^{2}}{A^{2}}\right)$
(d)

The time period of oscillation of a spring does not depend on gravity
(a)

The standard equation in SHM is

$$
\begin{equation*}
x=a \cos (\omega t+\emptyset) \tag{i}
\end{equation*}
$$

Where $a$ is amplitude, $\omega$ the angular velocity and ( $\varnothing$ ) the phase difference.
Also, $\omega=\frac{2 \pi}{T}$ where $T$ is periodic time.
So, Eq. (i) becomes

$$
\begin{equation*}
x=a \cos \left(\frac{2 \pi t}{T}+\emptyset\right) \tag{ii}
\end{equation*}
$$

Given, equation is

$$
\begin{equation*}
x=0.01 \cos \left(\frac{2 \pi t}{2}+\frac{\pi}{4}\right) \tag{iii}
\end{equation*}
$$

Comparing Eq. (ii) with Eq. (iii), we get

$$
\begin{aligned}
& \frac{2 \pi t}{T}=\frac{2 \pi t}{2} \\
\Rightarrow & T=2 \mathrm{~s}
\end{aligned}
$$

So, frequency $n=\frac{1}{T}=\frac{1}{2}=0.5 \mathrm{~Hz}$
(a)

When lift falls freely effective acceleration and frequency of oscillations become zero $g_{\text {eff }}=0 \Rightarrow T^{\prime}=\infty$, hence a frequency $=0$
(d)

When the force $F$ is applied to one side of block $A$, let the upper face of $A$ be displaced through distance $\Delta L$
Then

$\eta=\frac{F / L^{2}}{\Delta L / L}$ or $F=\eta L \Delta L$
So, $F \propto \Delta L$ and this force is restoring one. So, if the force is removed, the block will execute SHM
From Eq. (i) spring factor $=\eta L$
Here, inertia factor $=M$
$\therefore$ Time period, $T=2 \pi \sqrt{\frac{M}{\eta L}}$
(d)
$\frac{T^{\prime}}{T}=\sqrt{\frac{\mathrm{g}}{\mathrm{g}^{\prime}}}=\sqrt{\left[\frac{\mathrm{g}}{\mathrm{g}\left(1-\frac{1}{10}\right)}\right]}=\sqrt{\frac{10}{9}}$
$\Rightarrow \quad T^{\prime}=\sqrt{\frac{10}{9}} T$
(b)
$\frac{d^{2} x}{d t^{2}}=-\alpha x$
We know,

$$
\begin{equation*}
a=\frac{d^{2} y}{d t^{2}}=-\omega^{2} x \tag{ii}
\end{equation*}
$$

From Eqs.(i) and (ii), we have

$$
\begin{array}{cc} 
& \omega^{2}=\alpha \\
& \omega=\sqrt{\alpha} \\
\text { or } & \frac{2 \pi}{T}=\sqrt{\alpha} \\
\therefore & T=\frac{2 \pi}{\sqrt{\alpha}}
\end{array}
$$

(c)

Time period

$$
\because \quad m g=k x
$$

$$
\therefore \quad T=2 \pi \sqrt{\frac{x}{\mathrm{~g}}}
$$

$$
\begin{gathered}
T=2 \pi \sqrt{\frac{m}{k}} \\
m \mathrm{~g}=k x \\
T=2 \pi \sqrt{\frac{x}{\mathrm{~g}}} \\
(0.5)^{2}=4 \pi^{2} \times \sqrt{\frac{x}{10}} \\
\frac{(0.5)^{2} \times 9.8}{4 \times 3.14 \times 3.14}=x \\
x=0.0621 \mathrm{~m} \\
x=6.2 \mathrm{~cm}
\end{gathered}
$$

(a)

Given that, the time period of particle $A=T$ and the time period of particle $B=\frac{5 T}{4}$
Hence, the time difference $(\Delta T)=\frac{5 T}{4}-T$
$\Rightarrow \quad \Delta T=\frac{T}{4}$
The relation between phase difference and time difference is

$$
\begin{gathered}
\Delta \emptyset=\frac{2 \pi}{T} \Delta T \\
\Delta \emptyset=\frac{2 \pi}{T} \times \frac{T}{4} \\
\Rightarrow \quad \Delta \emptyset=\frac{\pi}{2}
\end{gathered}
$$

(c)

As $X$ and $Y$ have negligible mass, both the spring balances read the same force 8 kg or 8 kg (c)

If a spring of spring constant $k$ is divided into $n$ equal parts, the spring constant of each part becomes $n k$. So, effective spring constant

$$
\begin{aligned}
k & =k_{1}+k_{2} \\
& =4 k+4 k=8 k
\end{aligned}
$$

(c)


As springs and supports ( $M_{1}$ and $M_{2}$ ) are having negligible mass. Whenever springs pull the massless supports, springs will be in natural length. At maximum compression, velocity of $B$ will be zero


And by energy conservation
$\frac{1}{2}(4 K) y^{2}=\frac{1}{2} K x^{2} \Rightarrow \frac{y}{x}=\frac{1}{2}$
(a)
$A_{\text {max }}=\omega^{2} a$
(c)

Let displacement equation of particle executing SHM is
$y=a \sin \omega t$
As particle travels half of the amplitude from the equilibrium position, so $y=\frac{a}{2}$
Therefore, $\frac{a}{2}=a \sin \omega t \Rightarrow \sin \omega t=\frac{1}{2}=\sin \frac{\pi}{6}$
$\Rightarrow \omega t=\frac{\pi}{6} \Rightarrow t=\frac{\pi}{6 \omega} \Rightarrow t=\frac{\pi}{6\left(\frac{2 \pi}{T}\right)}\left(\operatorname{As} \omega=\frac{2 \pi}{T}\right)$
$\Rightarrow t=\frac{T}{12}$
Hence, the particle travels half of the amplitude from the equilibrium in $\frac{T}{12} S$
(d)

Let $r$ be the amplitude of oscillation and $T$ be the time period in SHM. Then total distance travelled in time $T=4 r$
$\therefore$ Average velocity, $v_{a v}=\frac{4 t}{T}=\frac{4 r}{2 \pi / \omega}$
$=\frac{2 r \omega}{\pi}=\frac{2 v_{\max }}{\pi}$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |
| A. | B | A | A | C | D | A | A | D | D | B |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |  |
| A. | C | A | C | B | C | C | A | A | C | D |  |  |  |
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