CLASS : XITh
Solutions

## Topic:- OSCILLATIONS

1
(c)

The motion of sphere is simple harmonic. It's time period ( $T_{0}$ ) is given by

where $l$ is length of string, $g$ the acceleration due to gravity.
When sphere is placed in electric field. ( $E$ ) force due to electric field acts on the sphere-1

$$
F_{E}=q E=m \mathrm{~g}
$$

where $q$ is charge on sphere.
Hence, resultant acceleration is

$$
\begin{array}{lll} 
& \mathrm{g}^{\prime}=\mathrm{g}+\frac{q E}{m} \\
\therefore & & T=2 \pi \sqrt{\frac{l}{\mathrm{~g}+\frac{q E}{m}}} \tag{ii}
\end{array}
$$

[Time period decreases]
Dividing Eq. (ii) by Eq. (i), we get

$$
\frac{T}{T_{0}}=\sqrt{\frac{\mathrm{g}}{\mathrm{~g}+\frac{q E}{m}}}
$$

2
(c)

The time period of simple pendulum in air

$$
\begin{equation*}
T=t_{0}=2 \pi \sqrt{\left(\frac{l}{\mathrm{~g}}\right)} \tag{i}
\end{equation*}
$$

$l$, being the length of simple pendulum.
In water, effective weight of bob
$w^{\prime}=$ weight of bob in air - upthrust
$\Rightarrow \quad \rho V g_{\text {eff }}=m g-m^{\prime} g$

$$
=\rho V g-\rho^{\prime} V g=\left(\rho-\rho^{\prime}\right) V g
$$

where $\quad \rho^{\prime}=$ density of bob,

$$
\rho=\text { density of water }
$$

$$
\therefore \quad g_{\text {eff }}=\left(\frac{\rho-\rho^{\prime}}{\rho}\right) g=\left(1-\frac{\rho^{\prime}}{\rho}\right) g
$$

$$
\therefore \quad t=2 \pi \sqrt{\left[\frac{l}{\left(1-\frac{\rho}{\rho}\right) \mathrm{g}}\right]}
$$

Thus, $\quad \frac{t}{t_{0}}=\sqrt{\left[\frac{1}{\left(1-\frac{\rho^{\prime}}{\rho}\right)}\right]}$

$$
=\sqrt{\left(\frac{1}{1-\frac{1000}{(4 / 3 \times 1000)}}\right)}=\sqrt{\left(\frac{4}{4-3}\right)}=2
$$

$$
\Rightarrow \quad t=2 t_{0}
$$

(b)

PE varies from zero to maximum. It is always positive sinusoidal function
(a)

Let $T_{1}, T_{2}$ be the time period of shorter length and longer length pendulums respectively.
Ads per question, $n T_{1}=(n-1) T_{2}$;
So $n 2 \pi \sqrt{\frac{0.5}{\mathrm{~g}}}=(n-1) 2 \pi \sqrt{\frac{20}{\mathrm{~g}}}$
or $n=(n-1) \sqrt{40} \approx(n-1) 6$
Hence, $5 n=6$
Hence, after 5 oscillations they will be in same phase
(c)

At centre $v_{\max } \Rightarrow a \omega=a \cdot \frac{2 \pi}{T}=\frac{0.2 \times 2 \pi}{0.01}=40 \pi$
(d)
$F_{1}=\frac{m 4 \pi^{2} a}{\pi^{2}}$ and $F_{2}=\frac{m 4 \pi^{2} a}{T_{2}^{2}}$
$F=F_{1}+F_{2}=\frac{4 \pi^{2} m a}{T_{1}^{2}}+\frac{4 \pi^{2} m a}{T_{2}^{2}}$
$=4 \pi^{2} m a\left(\frac{1}{T_{1}^{2}}+\frac{1}{T_{2}^{2}}\right)$
Or $\frac{4 \pi^{2} m a}{T^{2}}=4 \pi^{2} m a\left(\frac{1}{T_{1}^{2}}+\frac{1}{T_{2}^{2}}\right)$
Or $\frac{1}{T^{2}}=\frac{1}{T_{1}^{2}}+\frac{1}{T_{2}^{2}}$
Or $\frac{1}{T^{2}}=\frac{T_{1}^{2}+T_{2}^{2}}{T_{1}^{2} T_{2}^{2}}$ or $T^{2}=\frac{T_{1}^{2} T_{2}^{2}}{T_{1}^{2}+T_{2}^{2}}$
(b)
$T=2 \pi \sqrt{\frac{l}{g}} \Rightarrow l \propto T^{2}$ [Equation of parabola]
(d)

Here, $m=4 \mathrm{~kg} ; k=800 \mathrm{Nm}^{-1} ; E=4 \mathrm{~J}$
In SHM, total energy is $E=\frac{1}{2} k A^{2}$
where $A$ is the amplitude of oscillation
$\therefore 4=\frac{1}{2} \times 800 \times A^{2}$
$A^{2}=\frac{8}{800}=\frac{1}{100}$
$\Rightarrow A=\frac{1}{10} m=0.1 m$
Maximum acceleration, $a_{\max }=\omega^{2} A$
$=\frac{k}{m} A \quad\left[\because \omega=\sqrt{\frac{k}{m}}\right]$
$=\frac{800 \mathrm{Nm}^{-1}}{4 \mathrm{~kg}} \times 0.1 \mathrm{~m}=20 \mathrm{~ms}^{-2}$
(b)


Force constant $(k) \propto \frac{1}{\text { Length of spring }}$
$\Rightarrow \frac{K}{K_{1}}=\frac{l_{1}}{l}=\frac{\frac{2}{3} l}{l} \Rightarrow K_{1}=\frac{3}{2} K$
(b)

Total energy $U=\frac{1}{2} K a^{2}$
(b)
$\omega=\sqrt{k / m}=\sqrt{\frac{4.84}{0.98}}=2.22 \mathrm{rad} / \mathrm{s}$
(a)

For resonance amplitude must be maximum which is possible only when the denominator of expansion is zero
i.e. $a \omega^{2}-b \omega+c=0 \Rightarrow \omega=\frac{+b \pm \sqrt{b^{2}-4 a c}}{2 a}$

For a single resonant frequency, $b^{2}=4 a c$
(a)

Inside the mine $g$ decreases
Hence from $T=2 \pi \sqrt{\frac{l}{g}} ; T$ increase
(c)
$T=2 \pi \sqrt{\frac{l}{g}} \Rightarrow \frac{l}{T^{2}}=\frac{g}{4 \pi^{2}}=$ constant
(a)

KE of a body undergoing SHM is given by
$\mathrm{KE}=\frac{1}{2} m \omega^{2} A^{2} \cos ^{2} \omega t$ and $\mathrm{KE}_{\max }=\frac{m \omega^{2} A^{2}}{2}$
[symbols represent standard quantities]
From given information

$$
\begin{array}{ll} 
& \mathrm{KE}=\left(\mathrm{KE}_{\max }\right) \times \frac{75}{100} \\
\Rightarrow & \frac{m \omega^{2} A^{2}}{2} \cos ^{2} \omega t=\frac{m \omega^{2} A^{2}}{2} \times \frac{3}{4} \\
\Rightarrow & \cos \omega t= \pm \frac{\sqrt{3}}{2} \\
\Rightarrow \quad & \omega t=\frac{\pi}{6} \\
\Rightarrow \quad & \frac{2 \pi}{T} \times t=\frac{\pi}{6} \\
\Rightarrow \quad & t=\frac{T}{12}=\frac{1}{6} \mathrm{~s}
\end{array}
$$

(d)

When spring is cut into two equal parts then spring constant of each part will be $2 K$ and so using $n \propto \sqrt{K}$, new frequency will be $\sqrt{2}$ times, i.e. $f_{2}=\sqrt{2} f_{1}$
(a)

$$
\begin{aligned}
& \text { Time period of pendulum } T=2 \pi \sqrt{\frac{l}{\mathrm{~g}}} \\
& \therefore
\end{aligned}
$$

(d)

Let $x$ be the point where K.E. $=$ P.E.
Hence $\frac{1}{2} m \omega^{2}\left(a^{2}-x^{2}\right)=\frac{1}{2} m \omega^{2} x^{2}$
$\Rightarrow 2 x^{2}=a^{2} \Rightarrow \frac{\mathrm{a}}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2 \sqrt{2} \mathrm{~cm}$
(d)

The periodic time of a simple pendulum is given by,

$$
T=2 \pi \sqrt{\frac{l}{\mathrm{~g}}}
$$

When taken to height $2 R$.

$$
\begin{aligned}
& \mathrm{g}^{\prime}=\mathrm{g}\left(1+\frac{\mathrm{h}}{R_{e}}\right)^{2} \\
& =\mathrm{g}\left(1+\frac{2 R}{R}\right)^{-2}=\mathrm{g}(3)^{-2} \\
& \therefore \quad \frac{T_{1}}{T_{2}}=\sqrt{\frac{1}{3^{2}}} \\
& \Rightarrow \quad T_{2}=3 T_{1} \Rightarrow \frac{T_{1}}{T_{2}}=\frac{1}{3}
\end{aligned}
$$

(d)

$T \sin \theta=m L \sin \theta \omega^{2}$
$324=0.5 \times 0.5 \times \omega^{2}$
$\Rightarrow \omega^{2}=\frac{324}{0.5 \times 0.5}$
$\Rightarrow \omega=\sqrt{\frac{324}{0.5 \times 0.5}}$
$\Rightarrow \omega=\frac{18}{0.5}=36 \mathrm{rad} / \mathrm{sec}$


| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| A. | C | C | B | A | C | D | B | D | B | B |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |
| A. | B | A | A | C | A | D | A | D | D | D |  |  |
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