

Topic :- MOTION IN A STRAIGHT LINE

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(a)

Distance covered in 5th second

$$S_{5^{\text{th}}} = u + \frac{a}{2}(2n - 1) = 0 + \frac{a}{2}(2 \times 5 - 1) = \frac{9a}{2}$$

and distance covered in 5 second,

$$S_5 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times a \times 25 = \frac{25a}{2}$$

$$\therefore \frac{S_{5^{\text{th}}}}{S_5} = \frac{9}{25}$$

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(a)

$$S \propto u^2 \therefore \frac{S_1}{S_2} = \left(\frac{u_1}{u_2}\right)^2 \Rightarrow \frac{2}{S_2} = \frac{1}{4} \Rightarrow S_2 = 8 \text{ m}$$

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(c)

$$\frac{dx}{dt} = 2at - 3bt^2 \Rightarrow \frac{d^2x}{dt^2} = 2a - 6bt = 0 \Rightarrow t = \frac{a}{3b}$$

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(c)

Distance travelled by the particle is

$$x = 40 + 12t - t^3$$

We know that, speed is rate of change of distance i.e.

$$v = \frac{dx}{dt}$$

$$\therefore v = \frac{d}{dt}(40 + 12t - t^3) = 0 + 12 - 3t^2$$

But final velocity $v = 0$

$$\therefore 12 - 3t^2 = 0$$

$$\Rightarrow t^2 = \frac{12}{3} = 4 \Rightarrow t = 2 \text{ s}$$

Hence, distance travelled by the particle before coming to rest is given by

$$x = 40 + 12(2) - (2)^3 = 40 + 24 - 8 = 64 - 8 = 56 \text{ m}$$

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(c)

From equation of motion, we have

$$s = ut + \frac{1}{2}gt^2$$

Where, u is initial velocity, g the acceleration due to gravity and t the time.

For upward motion

$$h = -ut_1 - \frac{1}{2}gt_1^2 \quad \dots(i)$$

for downward motion

$$h = -ut_2 + \frac{1}{2}gt_2^2 \quad \dots(ii)$$

multiplying Eq. (i) by t_2 and Eq. (ii) by t_1 and subtracting Eq. (ii) by Eq. (i), we get

$$h(t_2 - t_1) = \frac{1}{2}gt_1t_2(t_2 - t_1)$$

$$h = \frac{1}{2}gt_1t_2 \quad \dots(iii)$$

When stone is dropped from rest $u = 0$, reaches the ground in t second.

$$\therefore h = \frac{1}{2}gt^2 \quad \dots(iv)$$

Equating Eqs. (iii) and (iv), we get

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2$$

$$\Rightarrow t^2 = t_1t_2 \Rightarrow t = \sqrt{t_1t_2}$$

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(c)

$$\frac{dv}{dt} = 6t \text{ or } dv = 6t, mv = \frac{6t^2}{2} = 3t^2,$$

$$dx = 3t^2 dt \Rightarrow x = 3 \frac{t^3}{2} = \frac{3t^3}{2}$$

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(b)

Region OA shows that graph bending toward time axis *i.e.* acceleration is negative.

Region AB shows that graph is parallel to time axis *i.e.* velocity is zero. Hence acceleration is zero.

Region BC shows that graph is bending towards displacement axis *i.e.* acceleration is positive.

Region CD shows that graph having constant slope *i.e.* velocity is constant. Hence acceleration is zero

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(a)

$$s \propto t^2 [\text{Given}] \therefore s = Kt^2$$

$$\text{Acceleration } a = \frac{d^2s}{dt^2} = 2k \text{ [constant]}$$

It means the particle travels with uniform acceleration

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(c)

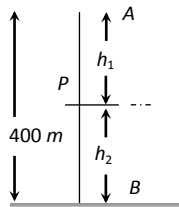
$$v^2 = u^2 + 2gh \Rightarrow v = \sqrt{u^2 + 2gh}$$

So for both the cases velocity will be equal

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(c)

Let both balls meet at point P after time t



The distance travelled by ball A, $h_1 = \frac{1}{2}gt^2$

The distance travelled by ball B, $h_2 = ut - \frac{1}{2}gt^2$

$$h_1 + h_2 = 400 \text{ m} \Rightarrow ut = 400, t = 400/50 = 8 \text{ sec}$$

$$\therefore h_1 = 320 \text{ m and } h_2 = 80 \text{ m}$$

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(c)

$$v = (180 - 16x)^{1/2}$$

$$\text{As } a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$\therefore a = \frac{1}{2}(180 - 16x)^{-1/2} \times (-16) \left(\frac{dx}{dt} \right)$$

$$= -8(180 - 16x)^{-1/2} \times v$$

$$= -8(180 - 16x)^{-1/2} \times (180 - 16x)^{1/2} = -8 \text{ m/s}^2$$

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(a)

Slope of displacement time-graph is velocity

$$\frac{v_1}{v_2} = \frac{\tan \theta_1}{\tan \theta_2} = \frac{\tan 30^\circ}{\tan 45^\circ} = \frac{1}{\sqrt{3}}$$

$$v_1 : v_2 = 1 : \sqrt{3}$$

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(a)

The $v - x$ equation from the given graph can be written as,

$$v = \left(-\frac{v_0}{x_0} \right) x + v_0 \quad \dots(i)$$

$$\therefore a = \frac{dv}{dt} = \left(-\frac{v_0}{x_0}\right) \frac{dx}{dt} = \left(-\frac{v_0}{x_0}\right) v$$

Substituting v from Eq. (i), we get

$$a = \left(-\frac{v_0}{x_0}\right) \left[\left(-\frac{v_0}{x_0}\right) x + v_0\right]$$

$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus, $a - x$ graph is a straight line with positive slope and negative intercept.

16 **(a)**

When the stone is released from the balloon. Its height

$$h = \frac{1}{2}at^2 = \frac{1}{2} \times 1.25 \times (8)^2 = 40 \text{ m and velocity}$$

$$v = at = 1.25 \times 8 = 10 \text{ m/s}$$

Time taken by the stone to reach the ground

$$t = \frac{v}{g} \left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right] = \frac{10}{10} \left[1 + \sqrt{1 + \frac{2 \times 10 \times 40}{(10)^2}}\right] = 4 \text{ sec}$$

17 **(c)**

Let v_1, v_2 be the initial speeds of first and second runners. Let t be time by them when the first runner has completed 50m. During this time, the second runner has covered a distance = 50 - 1 = 49m.

$$\text{So, } t = \frac{50}{v_1} = \frac{49}{v_2} \quad \dots(i)$$

Suppose, the second runner increases his speed to v_3 so that he covers the remaining distance (= 51m) in times t . So

$$t = \frac{51}{v_3} = \frac{49}{v_2}$$

$$\text{or } v_3 = \frac{51}{49}v_2$$

$$\text{or } v_3 = \left(1 + \frac{2}{49}\right)v_2 \quad \text{or} \quad \frac{v_3}{v_2} - 1 = \frac{2}{49}$$

$$\text{or } \frac{v_3 - v_2}{v_2} = \frac{2}{49}$$

$$\text{or } \% \text{ increase} = \frac{2}{49} \times 100 = 4.1\%$$

18 **(a)**

If t_0 is the reaction time, then the distance covered during decelerated motion is $10 - 10t_0$.

Now, in the first case,

$$10^2 = 2a(10 - 10t_0) \quad \dots(i)$$

Similarly, in the second case,

$$20^2 = 2a(30 - 20t_0) \quad \dots(ii)$$

Again, in the third case,

$$15^2 = 2a(x - 5t_0) \quad \dots(\text{iii})$$

$$\text{Dividing Eq.(ii) by Eq. (i), } \frac{20^2}{10^2} = \frac{30 - 20t_0}{10 - 10t_0}$$

$$\text{or } 40 - 40t_0 = 30 - 20t_0$$

$$\text{or } 20t_0 = 10 \text{ or } t_0 = \frac{1}{2}\text{s}$$

Dividing Eq. (iii) by Eq. (i), we get

$$\frac{225}{100} = \frac{x - 15t_0}{10 - 10t_0} \text{ or } \frac{9}{4} = \frac{x - 15 \times \frac{1}{2}}{10 - 10 \times \frac{1}{2}}$$

$$45 = 4x - 30 \text{ or } 4x = 75$$

$$\text{or } x = \frac{75}{4}\text{m} = 18.75\text{m}$$

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(c)

$$\mathbf{u} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}, \mathbf{a} = 0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}}$$

Speed $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$$= 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + (0.4\hat{\mathbf{i}} + 0.3\hat{\mathbf{j}})10$$

$$= 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} = 7\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$$

$$v = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ unit}$$

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(a)

If t_1 and $2t_2$ are the time taken by particle to cover first and second half distance respectively

$$t_1 = \frac{x/2}{3} = \frac{x}{6} \quad \dots(\text{i})$$

$$x_1 = 4.5 t_2 \text{ and } x_2 = 7.5 t_2$$

$$\text{So, } x_1 + x_2 = \frac{x}{2} \Rightarrow 4.5 t_2 + 7.5 t_2 = \frac{x}{2}$$

$$t_2 = \frac{x}{24} \quad \dots(\text{ii})$$

$$\text{Total time } t = t_1 + 2t_2 = \frac{x}{6} + \frac{x}{12} = \frac{x}{4}$$

So, average speed = 4 m/sec

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	A	A	C	C	C	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	C	A	A	A	C	A	C	A

PE