

Topic :- MECHANICAL PROPERTIES OF SOLIDS

1

(b)

$$E = \frac{1}{2} \frac{YA/\Delta l^2}{l}$$

$$\text{But } m = Ald \text{ or } A = \frac{m}{ld}$$

$$\therefore E = \frac{Ym\Delta l^2}{2l^2d}$$

$$E \text{ in calorie} = \frac{Ym\Delta l^2}{2l^2d}$$

$$\text{Now, } mS\theta = \frac{Ym\Delta l^2}{2l^2dJ} \text{ or } \theta = \frac{Y\Delta l^2}{2l^2dJS}$$

$$\text{Or } \theta = \frac{12 \times 10^{11} \times 10^{-1} \times 10^{-3} \times 10^{-3}}{2 \times 2 \times 2 \times 9 \times 10^3 \times 4.2 \times 0.1 \times 10^3}$$
$$= \frac{12 \times 10^5}{72 \times 42 \times 10^5} = \frac{1}{252} ^\circ\text{C}$$

2

(a)

$$l = \frac{FL}{\pi r^2 Y} \therefore l \propto \frac{L}{r^2} [Y \text{ and } F \text{ are constant}]$$

$$\frac{l_2}{l_1} = \frac{L_2}{L_1} \times \left(\frac{r_1}{r_2}\right)^2 = (2) \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\Rightarrow l_2 = \frac{l_1}{2} = \frac{0.01m}{2} = 0.005m$$

3

(d)

$$\text{Stress} = \frac{\text{Force}}{\text{area}}$$

In the present case, force applied and area of cross-section of wires are same, therefore stress has to be the same

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

Since the Young's modulus of steel wire is greater than the copper wire, therefore, strain in case of steel wire is less than that in case of copper wire

5

(a)

$$\eta = \frac{F}{A\theta} = \frac{5 \times 10^5}{100 \times 10^{-4} \times 0.001} = 5 \times 10^{10} \text{Nm}^{-2}$$

6 **(a)**

$$\frac{dV}{V} = (1 + 2\sigma) \frac{dL}{L}$$
 If $\sigma = -\frac{1}{2}$ then $\frac{dV}{V} = 0$ i.e. $K = \infty$

7 **(d)**
 Poisson's ratio is 0.5 so there is no change in the volume.

8 **(a)**

$$Y = \frac{FL}{Al} = \frac{1000 \times 100}{10^{-6} \times 0.1} = 10^{12} \text{N/m}^2$$

9 **(d)**

$$K = \frac{p}{-\frac{\Delta V}{V}} \Rightarrow K = \frac{h\rho g}{0.1 \times 10^{-2}}$$

$$\Rightarrow h = \frac{K \times 0.1 \times 10^{-2}}{\rho g} = \frac{9 \times 10^8 \times 10^3}{10^3 \times 10} = 90 \text{ m}$$

10 **(c)**
 Increase in length $l = \frac{FL}{AY}$
 or $l = \frac{FR^2}{\pi r^2 Y}$
 Percent increase in length

$$\Delta x = \frac{l}{L} \times 100 = \frac{F}{\pi r^2 Y}$$

 Here, same longitudinal force is applied.
 So, $\frac{\Delta x_1}{\Delta x_2} = \left(\frac{r_2}{r_1}\right)^2 \cdot \left(\frac{Y_2}{Y_1}\right)$

$$\frac{1}{\Delta x_2} = \left(\frac{1}{2}\right)^2 \cdot \left(\frac{2}{1}\right) = \frac{1}{4} \times \frac{2}{1}$$

$$\frac{1}{\Delta x_2} = \frac{1}{2}$$

$$\Delta x_2 = 1 \times 2 = 2\%$$

11 **(a)**
 $F = YA\alpha T$;

$$\frac{F_{Cu}}{F_{Fe}} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{3}{2}$$

13 **(b)**
 At point b, yielding of material starts

14 **(c)**
 Restoring force is zero at mean position

$$F = -Kx + F_0 \Rightarrow 0 = -Kx + F_0 \Rightarrow x = \frac{F_0}{K}$$

i.e. the particle will oscillate about $x = \frac{F_0}{K}$

$$\Rightarrow F_0 = Kx \Rightarrow ma = Kx \Rightarrow a = \frac{K}{m} x \therefore W = \sqrt{\frac{K}{m}}$$

15 **(b)**

$$\text{Strain} \propto \text{Stress} \propto \frac{F}{A}$$

$$\text{Ratio of strain} = \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$$

16 **(b)**

$$\frac{1}{K} = \frac{\Delta V/V}{\Delta p} \quad \text{or} \quad \frac{\Delta V}{V} = \Delta p \left[\frac{1}{K}\right]$$

$$\text{Or } \frac{\Delta V}{V} \times 100 = 10^5 \times 8 \times 10^{-12} \times 100 = 8 \times 10^{-5}$$

17 **(a)**

$$Y = \frac{F/A}{\text{Strain}} \Rightarrow \text{strain} = \frac{F}{AY}$$

18 **(b)**

$$F = -\left(\frac{dU}{dx}\right)$$

In the region *BC* slope of the graph is positive

$\therefore F =$ negative *i.e.* force is attractive in nature

In the region *AB* slope of the graph is negative

$\therefore F =$ positive *i.e.* force is repulsive in nature

19 **(a)**

Total work done in the stretching a string

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

Hence, the work done per unit volume is $\frac{1}{2}(\text{stress} \times \text{strain})$.

This work is stored as the potential energy in the string.

20 **(b)**

$$Y = \frac{FL}{Al} = \frac{4FL}{\pi l^2 l}; F = mg$$

Where *L* = length of the wire

l = elongation of the wire

d = diameter of the wire

substituting the values, we get $Y = 2 \times 10^{11} \text{N/m}^2$

$$\Rightarrow \frac{\Delta Y}{Y} = 2 \frac{\Delta d}{d} + \frac{\Delta l}{l} = 2 \left(\frac{0.01}{0.4}\right) + \frac{0.05}{0.8} = \frac{9}{80}$$

$$\Rightarrow \Delta Y = \frac{9}{80} \times Y = \frac{9}{80} \times 2 \times 10^{11} = 0.2 \times 10^{11} \text{N/m}^2$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	D	C	A	A	D	A	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	B	C	B	B	A	B	A	B

PE