CLASS : XIth
Solutions

## Topic :- GRAVITATION

(b)

The energy required to remove the satellite from its orbit around the earth to infinity is called binding energy of the satellite. It is equal to negative of total mechanical energy of satellite in its orbit.
Thus, binding energy $=-E=\frac{G M m}{2 r}$
but,

$$
g=\frac{G M}{R^{2}}
$$

$\Rightarrow \quad G M=g R^{2}$
$\therefore \quad \mathrm{BE}=\frac{g m R^{2}}{2 r}$
(c)

Applying law of conservation of energy for asteroid at a distance $10 R_{e}$ and at earth's surface.


Now, $K_{f}=\frac{1}{2} m v_{i}^{2}$ and $U_{i}=-\frac{G M_{e} m}{10 R_{e}}$

$$
K_{f}=\frac{1}{2} m v_{f}^{2} \text { and } U_{f}=-\frac{G M_{e} m}{R_{e}}
$$

Substituting these values in Eq. (i), we get

$$
\begin{aligned}
& \frac{1}{2} m v_{i}^{2}-\frac{G M_{e} m}{10 R_{e}}=\frac{1}{2} m v_{f}^{2}-\frac{G M_{e} m}{R_{e}} \\
\Rightarrow & \frac{1}{2} m v_{f}^{2}=\frac{1}{2} m v_{f}^{2}+\frac{G M_{e} m}{R_{e}}-\frac{G M_{e} m}{10 R_{e}} \\
\Rightarrow & v_{f}^{2}=v_{i}^{2}+\frac{2 G M_{e}}{R_{e}}-\frac{2 G M_{e}}{10 R_{e}} \\
\therefore \quad & v_{f}^{2}=v_{i}^{2}+\frac{G M_{e} m}{R_{e}}\left(1-\frac{1}{10}\right)
\end{aligned}
$$

(b)
$V=-\int_{\infty}^{x} I d x=-\int_{\infty}^{x} \frac{C}{x^{2}} d x=\frac{C}{x}$
(c)
$U=$ Loss in gravitational energy = gain in K.E.
So, $U=\frac{1}{2} m v^{2} \Rightarrow m=\frac{2 U}{v^{2}}$

## (b)

$v_{e}=\sqrt{2} v_{o}$, i.e. if the orbital velocity of moon is increased by factor of $\sqrt{2}$ then it will escape out from the gravitational field of earth
(b)
$v_{e}=R \sqrt{\frac{8}{3}} G \pi \rho \quad \therefore v_{e} \propto R \sqrt{\rho}$
(c)
$\frac{T_{1}}{T_{2}}=\left(\frac{R_{1}}{R_{2}}\right)^{3 / 2}=\left(\frac{10^{13}}{10^{12}}\right)^{3 / 2}=(1000)^{1 / 2}=10 \sqrt{10}$
(b)
$F=\frac{G m\left(M_{-} m\right)}{x^{2}}$; For maxima,
$\frac{d F}{d m}=\frac{G}{x^{2}}(M-2 m)=0$
or $\frac{m}{M}=\frac{1}{2}$
(c)

Acceleration due to gravity on moon $g_{m}=\frac{G \times M / 90}{(R / 3)^{2}}=\frac{1}{10} g$
(c)

Acceleration due to gravity at poles is independent of the angular speed of earth
(a)

The change in potential energy in gravitational field is given by $\Delta E=G M m\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)$
In this problem; $r_{1}=R$ and $r_{2}=n R$
$\Delta E=\operatorname{GMm}\left(\frac{1}{R}-\frac{1}{n R}\right)$
$=\frac{G M m}{R}\left(\frac{n-1}{n}\right)$
$=m g R\left(\frac{n-1}{n}\right)\left(\because \mathrm{g}=\frac{G m}{R^{2}}\right)$
(d)

Let escape velocity be $v_{e}$, then kinetic energy is

$$
\begin{equation*}
=\frac{1}{2} m v_{e}^{2} \tag{i}
\end{equation*}
$$

and escape energy $=+\frac{G M_{\mathrm{e}} m}{R_{e}}$
Equating Eqs. (i) and (ii), we get

$$
\begin{array}{cc} 
& \frac{1}{2} m_{e}^{2}=\frac{G M_{\mathrm{e}} m}{R_{e}} \\
\Rightarrow & v_{e}=\sqrt{\frac{2 G M_{e}}{R_{e}}} \\
\Rightarrow & R=\frac{2 G M_{e}}{v_{e}^{2}} \\
\text { Given, } & G=6.67 \times 10^{-11} \mathrm{~N}-\mathrm{m}^{2} / \mathrm{kg}, \\
& M_{e}=6 \times 10^{24} \mathrm{~kg}, v_{\mathrm{e}}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}^{2} \\
& R=\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{\left(3 \times 10^{8}\right)^{2}} \\
& R=8.89 \times 10^{-3} \\
R \approx 9 \times 10^{-3} \mathrm{~m}=9 \mathrm{~mm}
\end{array}
$$

(d)

The body can be fired at any angle because the energy is sufficient to take the body out of the gravitational field of earth
(b)

Acceleration due to gravity at a height h from earth's surface

$$
\begin{aligned}
g^{\prime} & =\frac{G M}{(R+h)^{2}} \\
\text { Since, } \quad g^{\prime} & =\frac{g}{100} \\
\text { or } \quad \frac{g}{100} & =\frac{G M}{(R+h)^{2}} \\
\text { or } \quad \frac{(R+h)^{2}}{100} & =\frac{G M}{g}
\end{aligned}
$$

or $\quad \frac{(R+h)^{2}}{100}=R^{2} \quad\left[\therefore g=\frac{G M}{R^{2}}\right]$
or $\quad R+h=10 R$
$\Rightarrow \quad h=9 R$

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(d)

Acceleration due to gravity

$$
\left.\begin{array}{rl} 
& g
\end{array}=\frac{G M}{R^{2}}=\frac{G}{R^{2}} \times \frac{4}{3} \pi R^{3} \rho\right]
$$

(a)
$v_{e}=\sqrt{\frac{2 G M}{R}}=100 \Rightarrow \frac{G M}{R}=5000$
Potential energy $U=-\frac{G M m}{R}=-5000 \mathrm{~J}$
(c)
$g \propto r($ if $r<R)$ and $g \propto \frac{1}{r^{2}}($ if $r>R)$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |  |
| A. | B | A | C | A | B | C | B | B | C | C |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |  |  |
| A. | B | C | C | A | D | D | B | D | A | C |  |  |  |  |
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