CLASS : XIth
Solutions

## Topic :- GRAVITATION

(c)

Mass of two planets is same, so
$\frac{4}{3} \pi R_{1}^{3} \rho_{1}=\frac{4}{3} \pi R_{2}^{3} \rho_{2}$
or $\frac{R_{1}}{R_{2}}=\left(\frac{\rho_{2}}{\rho_{1}}\right)^{1 / 3}=\left(\frac{1}{8}\right)^{1 / 3}=\frac{1}{2}$
$\frac{\mathrm{g}_{1}}{\mathrm{~g}_{2}}=\frac{G M / R_{1}^{2}}{G M / R_{2}^{2}}=\left(\frac{R_{2}}{R_{1}}\right)^{2}=(2)^{2}=4$
(d)

Gravitational potential energy is given as

$$
\begin{aligned}
U & =-\frac{G M m}{r} \\
U_{1} & =-\frac{G M m}{r_{1}}, U_{2}=-\frac{G M m}{r_{2}}
\end{aligned}
$$

As $r_{2}>r_{1}$, hence,

$$
\begin{array}{cc} 
& U_{1}-U_{2}=G M m\left[\frac{r_{2}-r_{1}}{r_{1} r_{2}}\right] \text { is positie } \\
\text { ie, } & U_{1}>U_{2} \\
\text { or } & U_{2}<U_{1}
\end{array}
$$

$i e$, gravitational potential energy increases.
or $g=\frac{9 \times 21 \times 21}{20 \times 20}$
Now, $\quad \mathrm{g}_{d}=\mathrm{g}\left(1-\frac{d}{R}\right)$
$=\frac{9 \times 21 \times 21}{20 \times 20}\left[1-\frac{R / 20}{R}\right]=9.5 \mathrm{~ms}^{-2}$
(b)

Gravitational potential at a point on the surface of earth
$V=\frac{-G M}{R}=\frac{-\mathrm{g} R^{2}}{R}=-\mathrm{g} R$
(c)
$\frac{g^{\prime}}{g}=\left(\frac{R}{R+h}\right)^{2} \Rightarrow \frac{1}{2}=\left(\frac{R}{R+h}\right)^{2} \Rightarrow \frac{1}{2}=\left(\frac{4000}{4000+h}\right)^{2}$
By solving we get $\mathrm{h}=1656.85$ mile $\approx 1600$ mile
(d)

It is given that, acceleration due to gravity on planet $A$ is 9 times the acceleration due to gravity on planet $B$ ie,

$$
\begin{equation*}
g_{A}=9 g_{B} \tag{i}
\end{equation*}
$$

From third equation of motion

$$
\begin{equation*}
v^{2}=2 g h \tag{ii}
\end{equation*}
$$

At planet $A, h_{A}=\frac{v^{2}}{2 g_{A}}$
At planet $B, \mathrm{~h}_{B}=\frac{v^{2}}{2 g_{B}}$
Dividing Eq. (ii) by Eq. (iii), we have

$$
\frac{\mathrm{h}_{A}}{\mathrm{~h}_{B}}=\frac{g_{B}}{g_{A}}
$$

From Eq. (i), $g_{A}=9 g_{B}$

$$
\begin{array}{ccc}
\therefore & \frac{\mathrm{h}_{A}}{\mathrm{~h}_{B}}=\frac{g_{B}}{9 g_{B}}=\frac{1}{9} & \\
\text { or } & \mathrm{h}_{B}=9 \mathrm{~h}_{A}=9 \times 2=18 \mathrm{~m} & \left(\therefore \mathrm{~h}_{A}=2 \mathrm{~m}\right)
\end{array}
$$

(b)

Gravitational force provides the required centripetal force $i e$,

$$
m \omega^{2} R=\frac{G M m}{R^{\frac{5}{2}}}
$$

$$
\begin{array}{cc}
\Rightarrow & \frac{m 4 \pi^{2}}{T^{2}}=\frac{G M m}{R^{\frac{7}{2}}} \\
\Rightarrow & T^{2} \propto R^{7 / 2}
\end{array}
$$

(a)

Escape velocity, $v_{e}=\sqrt{\frac{2 G M_{e}}{R_{e}}}$
Given, $\quad M_{p}=6 M_{e}, R_{p}=2 R_{e}$
$\therefore \quad v_{p}=\sqrt{\frac{2 G \cdot 6 M_{e}}{\left(2 R_{e}\right)}}=\sqrt{3} v_{e}$
(d)
$T=2 \pi \sqrt{\frac{r^{3}}{G M}} \Rightarrow r^{3}=\frac{G M T^{2}}{4 \pi^{2}} \Rightarrow r=\left[\frac{G M T^{2}}{4 \pi^{2}}\right]^{1 / 3}$
(c)

If $M$ be mass of earth and $R$ its radius, the acceleration due to gravity is given by

$$
\begin{equation*}
g=\frac{G M}{R^{2}} \tag{i}
\end{equation*}
$$

Where, $G$ is gravitational constant.

$$
\begin{align*}
\text { Given, } & & R & =0.99 R \\
\therefore & & g^{\prime} & =\frac{G M}{(0.99 R)^{2}}  \tag{ii}\\
& & & =1.02\left(\frac{G M}{R^{2}}\right)
\end{align*}
$$

From Eq. (i), we get

$$
g^{\prime}=1.02 g
$$

Hence, acceleration due to gravity increases by

$$
g^{\prime}-g=1.02-1=0.02 g
$$

Hence, percentage increases $=2 \%$.

5 (c)
Acceleration due to gravity at a height h above the earth's surface is $g_{\mathrm{h}}=\frac{g}{\left(1+\frac{\mathrm{h}}{R}\right)^{2}}$
Where $g$ is the acceleration due to gravity on the earth's surface
At $\mathrm{h}=\frac{R}{2}, g_{\mathrm{h}}=\frac{g}{\left(1+\frac{R}{2 R}\right)^{2}}=\frac{4 g}{9}$
At $\mathrm{h}=R, g_{\mathrm{h}}=\frac{g}{\left(1+\frac{R}{R}\right)^{2}}=\frac{g}{4}$
Acceleration due to gravity at a depth $d$ below the earth's surface is $g_{d}=g\left(1-\frac{d}{R}\right)$
At $d=\frac{R}{2}, g_{d}=g\left(1-\frac{2}{2 R}\right)=\frac{g}{2}$
At the centre of earth, $d=R$
$g_{d}=g\left(1-\frac{R}{R}\right)=0$
Thus, the acceleration due to gravity is maximum on the earth's surface
(a)

As in case of elliptic orbit of a satellite mechanical energy
$E=-(G M m / 2 a)$ remains constant, at any position of satellite in the orbit,
$\mathrm{KE}+\mathrm{PE}=-\frac{G M m}{2 a}$
Now, if at position $r, v$ is the orbital speed of satellite
$\mathrm{KE}=\frac{1}{2} m v^{2}$ and $\mathrm{PE}=-\frac{G M m}{r}$
So, from Eqs. (i) and (ii), we have
$\frac{1}{2} m v^{2}-\frac{G M m}{r}=-\frac{G M m}{2 a}, i e, v^{2}=G M\left[\frac{2}{r}-\frac{1}{a}\right]$
(d)


Here, $O P=O Q=O R=\sqrt{2} \mathrm{~m}$
The gravitational force on mass 2 kg at $O$ due to mass
1 kg at $P$ is $F_{O P}=\frac{G \times 2 \times 1}{(\sqrt{2})^{2}}=G$ along $O P$
The gravitational force on mass 2 kg at $O$ due to mass
1 kg at $Q$ is $F_{O Q}=\frac{G \times 2 \times 1}{(\sqrt{2})^{2}}=G$ along $O Q$
The gravitational force on mass 2 kg at $O$ due to mass
1 kg at $R$ is $F_{O R}=\frac{G \times 2 \times 1}{(\sqrt{2})^{2}}=G$ along $O R$
Resolve forces $F_{O Q}$ and $F_{O R}$ into two rectangular components
$F_{O Q} \cos 30^{\circ}$ and $F_{O R} \cos 30^{\circ}$ are equal in magnitude of equal and opposite direction
$=F_{O P}-\left(F_{O Q} \sin 30^{\circ}+F_{O R} \sin 30^{\circ}\right)$
$=G-\left(G \times \frac{1}{2}+G \times \frac{1}{2}\right)=G-G=$ Zero $N$
(c)

Landsats 1 through 3 operated in a near polar orbit at an altitude of 920 km with an 18 day repeat coverage cycle. These satellites circled the earth every 103 min completing 14 orbits a day.
(d)

$$
\begin{aligned}
U_{i} & =-\frac{G M m}{r} \\
U_{i} & =\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1} \\
U_{i} & =-\frac{6.67 \times 10^{-11}}{0.1} \\
& =-6.67 \times 10^{-10} \mathrm{~J}
\end{aligned}
$$

We know

$$
\begin{aligned}
& \therefore \quad W=\Delta U \\
& =U_{f}-U_{i} \quad W=U_{i}=6.67 \times 10^{-10} \mathrm{~J} \\
& \therefore \quad\left(\therefore U_{f}=0\right) \\
& R=0.10 \times 10^{-3} \mathrm{~kg}
\end{aligned}
$$



| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |
| A. | C | D | A | B | B | A | A | C | D | B |  |  |  |
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| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |  |
| A. | B | A | D | C | C | A | D | C | D | D |  |  |  |
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