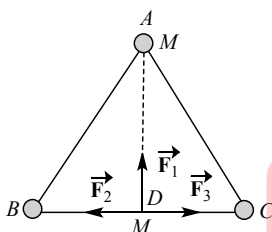


Topic :- GRAVITATION

1

(b)

(i) Gravitational force on the particle placed at mid point D of side BC of length a is



$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

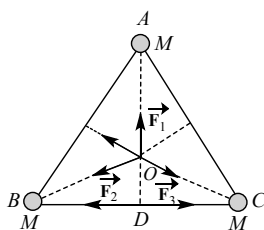
Here, $\vec{F}_2 = -\vec{F}_3$

$$\therefore \vec{F} = \vec{F}_1 + 0 = \vec{F}_1$$

$$\text{or } F = F_1 = \frac{GMM}{[AD]^2} = \frac{GM^2}{(3a^2/4)} = \frac{4GM^2}{3a^2}$$

(ii) gravitational force on the particle placed at the point O , i.e. the intersection of three medians is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \text{ or } F = 0$$



Since, the resultant of \vec{F}_2 and \vec{F}_3 is equal and opposite to \vec{F}_1

2

(b)

If g is the acceleration due to gravity of earth at the position of satellite, the apparent weight of a body in the satellite will be

$$W_{\text{app}} = m(g' - a)$$

But as satellite is a freely falling body, ie, $g' = a$

$$\text{So, } W_{\text{app}} = 0$$

3 **(d)**

$$\frac{K_A}{K_B} = \frac{r_B}{r_A} = \left(\frac{R + h_B}{R + h_A}\right) = \left(\frac{R + 2R}{R + R}\right) = \frac{3}{2}$$

4 **(b)**

$$\text{As mass, } M = \frac{4}{3}\pi R^2 \rho$$

$$\text{or } \rho = \frac{3M}{4\pi R^3}$$

$$\therefore \frac{\rho_s}{\rho_e} = \frac{M_s}{M_e} \times \frac{R_e^3}{R_s^3} = 330 \times \left(\frac{1}{100}\right)^3 = 3.3 \times 10^{-4}$$

5 **(c)**

$$U = \frac{-GMm}{r}, K = \frac{GMm}{2r} \text{ and } E = \frac{-GMm}{2r}$$

For a satellite U, K and E varies with r and also U and E remains negative whereas K remain always positive

6 **(a)**

$$g' = g - \frac{10g}{100} - \frac{90}{100}g$$

$$g' = g \frac{R^2}{(R+h)^2} \text{ or } \frac{9}{10} = \frac{R^2}{(R+h)^2}$$

$$\text{or } \frac{3}{\sqrt{10}} = \frac{R}{R+h}$$

$$\text{or } h = (\sqrt{10} - 3)R/3$$

$$\frac{(\sqrt{10} - 3) \times 6400}{3} = 345.60 \text{ km}$$

7 **(d)**

The minimum velocity of projection to achieve escape velocity can be calculated as

$$\begin{aligned} \text{Initial KE} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times m(4 \times 11.2)^2 = 16 \times \frac{1}{2}mv_e^2 \end{aligned}$$

As $\frac{1}{2}mv_e^2$ energy is used up in coming out from the gravitational pull of the earth, so

final KE should be $15 \times \frac{1}{2}mv_e^2$

$$\text{Hence, } \frac{1}{2}mv^2 = 15 \times \frac{1}{2}mv_e^2$$

$$\therefore v^2 = 15v_e^2$$

$$\begin{aligned} \text{or } v &= \sqrt{15}v_e \\ &= \sqrt{15} \times 11.2 \text{ kms}^{-1} \end{aligned}$$

8 (c)

$$\frac{T_{\text{mercury}}}{T_{\text{earth}}} = \left(\frac{r_{\text{mercury}}}{r_{\text{earth}}} \right)^{3/2} = \left(\frac{6 \times 10^{10}}{1.5 \times 10^{11}} \right)^{3/2} = \frac{1}{4}$$

(approx.)

$$\therefore T_{\text{mercury}} = \frac{1}{4} \text{ year}$$

9 (c)

$$\frac{g_m}{g_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m} \right) = \frac{1}{81} \times (4)^2 = \frac{16}{81}$$

$$g_m = \frac{16}{81} g_e$$

$$\therefore v_e = \sqrt{2g_e R_e} = \sqrt{2 \times 9.8 \times (6400 \times 1000)}$$
$$\approx 11.2 \text{ kms}^{-1}$$

$$v_m = \sqrt{2g_m R_m} = \sqrt{2 \times \frac{16}{81} g_e \times \frac{1}{4} R_e}$$
$$= \frac{2}{9} \sqrt{2g_e R_e} = \frac{2}{9} \times 11 \approx 2.5 \text{ kms}^{-1}$$

10 (a)

Acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

where G is gravitational constant.

$$\text{For earth: } g_e = \frac{GM_e}{R_e^2}$$

$$\text{For planet: } g_p = \frac{GM_p}{R_p^2}$$

$$\text{Therefore, } \frac{g_e}{g_p} = \frac{GM_e/R_e^2}{GM_p/R_p^2}$$

$$\text{or } \frac{g_e}{g_p} = \frac{M_e}{M_p} \times \frac{R_p^2}{R_e^2} \quad \dots(i)$$

Given, $M_p = 2M_e$, $R_p = 2R_e$

Putting the values in the Eq. (i), we obtain

$$\frac{g_e}{g_p} = \frac{M_e}{2M_e} \times \frac{(2R_e)^2}{R_e^2} = \frac{1}{2} \times \frac{4}{1} = 2$$

$$\therefore g_p = \frac{g_e}{2}$$

11 (c)

$v_e = \sqrt{\frac{2GM}{R}}$ i.e. escape velocity depends upon the mass and radius of the planet

12 (b)

When the spaceship is to take off, gravitational pull of earth requires more energy to be spent to overcome it

13 (d)

Acceleration due to gravity on earth is given by $g = \frac{GM}{R^2}$

$$\left(\text{Here, } M_m = \frac{M_e}{9}, R_m = \frac{R_e}{2} \right)$$

Hence,
$$\frac{g_e}{g_m} = \frac{M_e}{M_m} \times \frac{R_m^2}{R_e^2} = \frac{9M_e}{M_e} \times \left(\frac{R_e}{2R_e} \right)^2$$

or
$$\frac{g_e}{g_m} = \frac{9}{4}$$

So,
$$\frac{g_m}{g_e} = \frac{4}{9}$$

\therefore Weight of body on moon
 $=$ weight of body on earth $\times g_m/g_e$
 $= 90 \times \frac{4}{9} = 90 \times \frac{4}{9} = 40\text{kg}$

14 (b)

$$\omega_{\text{body}} = 27\omega_{\text{earth}}$$

$$T^2 \propto r^3 \Rightarrow \omega^2 \propto \frac{1}{r^3} \Rightarrow \omega \propto \frac{1}{r^{3/2}} \therefore r \propto \frac{1}{\omega^{2/3}}$$

$$\Rightarrow \frac{r_{\text{body}}}{r_{\text{earth}}} = \left(\frac{\omega_{\text{earth}}}{\omega_{\text{body}}} \right)^{2/3} = \left(\frac{1}{27} \right)^{2/3} = \frac{1}{9}$$

15 (d)

$$L = mvr = m \left(\sqrt{\frac{GM}{r}} \right) r = m\sqrt{GM}r \therefore L \propto \sqrt{r}$$

16 (c)

$$H = \frac{u^2}{2g} \Rightarrow H \propto \frac{1}{g} \Rightarrow \frac{H_B}{H_A} = \frac{g_A}{g_B}$$

Now $g_B = \frac{g_A}{12}$ as $g \propto \rho R$

$$\therefore \frac{H_B}{H_A} = \frac{g_A}{g_B} = 12$$

$$\Rightarrow H_B = 12 \times H_A = 12 \times 1.5 = 18\text{m}$$

17 (b)

$$T^2 \propto r^3$$

18 (c)

Force acting on a body of mass M at a point at depth d . Inside the earth is

$$F = mg' = mg \left(1 - \frac{d}{R} \right)$$

$$= \frac{mGM}{R^2} \left(\frac{R-d}{R} \right) = \frac{GMm}{R^3} r \quad (\because R-d=r)$$

So, $F \propto r$; Given $F \propto r^n$

$$n = 1$$

19 **(d)**

Let the gravitational force on a body mass m at O due to moon of mass M and earth of mass $81M$ be zero, where $EO = x$ and $MO = (r - x)$. Then,

$$\frac{G81M \times m}{x^2} = \frac{GMm}{(r-x)^2}$$

$$\text{or } \frac{81}{x^2} = \frac{1}{(r-x)^2}$$

$$\text{or } \frac{9}{x} = \frac{1}{(r-x)}$$

On solving; $x = 9r/10$

20 **(b)**

Gravitational force on a body at a distance x from the centre of earth $F = \frac{GMm}{x^2}$

Work done,

$$W = \int_R^{R+h} F dx = \int_R^{R+h} \frac{GMm}{x^2} dx$$

$$= GMm \left[-\frac{1}{x} \right]_R^{R+h} = mgR^2 \left[\frac{1}{R} - \frac{1}{R+h} \right]$$

This work done appears as increase in potential energy

$$\Delta E_p = mgR^2 \left[\frac{1}{R} - \frac{1}{R+h} \right]$$

$$= mg(5h)^2 \left[\frac{1}{5h} - \frac{1}{6h} \right] = \frac{5}{6} mgh$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	D	B	C	A	D	C	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	D	C	B	C	D	B

P E