Class: XIth
SUBJECT : PHYSICS
Date :

## Solutions

## Topic :- GRAVITATION

(a)

Time period, $T=\frac{2 \pi R}{\sqrt{\frac{G M_{m}}{R}}}=\frac{2 \pi R^{3 / 2}}{\sqrt{G M_{m}}}$
Where the symbols have their meaning as given in the question
Squaring both sides, we get
$T^{2}=\frac{4 \pi^{2} R^{3}}{G M_{m}}$
(d)

Orbital velocity $v_{0}=\sqrt{\frac{G M}{r}}=\sqrt{\frac{g R^{2}}{r}}$ and $v_{0}=r \omega$
This gives $r^{3}=\frac{R^{2} g}{\omega^{2}}$
(d)

Escape velocity $v_{e}=\sqrt{\frac{2 G M}{R}}$
$v_{e}=\sqrt{\frac{2 G \frac{4}{3} \pi R^{3} \times d}{R}}$
$\sqrt{2 G \frac{4}{3} \pi R^{3} \times d}=R \sqrt{\frac{8}{3} \pi G d}$
where $d=$ mean density of earth

$$
\begin{aligned}
& \because v_{e} \propto R \sqrt{d} \\
& \therefore \frac{v_{e}}{v_{p}}=\frac{R_{e}}{R_{p}} \sqrt{\frac{d_{e}}{d_{p}}} \\
& =\frac{R_{e}}{2 R_{e}} \sqrt{\frac{d_{e}}{d_{e}}} \\
& =v_{p}=2 v_{e} \\
& =2 \times 11=22 \mathrm{kms}^{-1}
\end{aligned}
$$

(d)

Here, $u=20 \mathrm{~ms}^{-1}, m=500 \mathrm{~g}=0.5 \mathrm{~kg}, t=20 \mathrm{~s}$

Using Newton's equation of motion

$$
\begin{aligned}
s & =u t+\frac{1}{2} g t^{2} \\
0 & =20 \times 20+\frac{1}{2}(-g)(20)^{2} \\
\text { or } \quad g & =2 \mathrm{~ms}^{-2}
\end{aligned}
$$

$\therefore$ Weight of body on planet $=m g$

$$
=0.5 \times 2=1 \mathrm{~N}
$$

(c)

Angular momentum remains constant
$m v_{1} d_{1}=m v_{2} d_{2} \Rightarrow v_{2}=\frac{v_{1} d_{1}}{d_{2}}$
(b)

As with height $g$ varies as
$\mathrm{g}^{\prime \prime}=\frac{\mathrm{g}}{[1+h / R]^{2}}=\mathrm{g}\left[1-\frac{2 h}{R}\right]$

and in according with figure $\mathrm{h}_{1}>\mathrm{h}_{2}$, so $W_{1}$ will be lesser than $W_{2}$ and
$W_{2}-W_{1}=m \mathrm{~g}_{2}-m \mathrm{~g}_{1}=2 m \mathrm{~g}\left[\frac{\mathrm{~h}_{1}}{R}-\frac{\mathrm{h}_{2}}{R}\right]$
or $W_{2}-W_{1}=2 m \frac{G M}{R^{2}} \frac{\mathrm{~h}}{R}$
$\left[\right.$ as $\mathrm{g}=\frac{G M}{R^{2}}$ and $\left.\left(\mathrm{h}_{1}-\mathrm{h}_{2}\right)=h\right]$
or $\quad W_{2}-W_{1}=\frac{2 m_{\mathrm{h}} G}{R^{3}}\left(\frac{4}{3} \pi R^{3} \rho\right)$
$=\frac{8}{3} \pi \rho G m \mathrm{~h}\left[\right.$ as $\left.M=\frac{4}{3} \pi R^{3} \rho\right]$
(c)

Time period of nearby satellite

$$
\begin{aligned}
T & =2 n \sqrt{\frac{r^{3}}{G M}} \\
& =2 \pi \sqrt{\frac{R^{3}}{G M}} \\
& =\frac{2 \pi\left(R^{3}\right)^{1 / 2}}{\left[G \frac{4}{3} \pi R^{3} \rho\right]^{1 / 2}}=\sqrt{\frac{3 \pi}{G \rho}}
\end{aligned}
$$

(a)

The acceleration due to gravity $(g)$ is given by

$$
g=\frac{G M}{R^{2}}
$$

where $M$ is mass, $G$ the gravitational constant and $R$ the radius.
Since, planets have a spherical shape

$$
V=\frac{4}{3} \pi r^{3}
$$

Also, $\quad \operatorname{mass}(M)=\operatorname{volume}(V) \times \operatorname{density}(\rho)$

$$
\begin{aligned}
g & =\frac{G \frac{4}{3} \pi R^{3} \rho}{R^{2}} \\
\Rightarrow \quad g & =\frac{4 G \pi \rho R}{3}
\end{aligned}
$$

Given, $\quad R_{1}: R_{2}=2: 3$

$$
\begin{aligned}
& \rho_{1}: \rho_{2}=\frac{3}{2} \\
\therefore & \frac{g_{1}}{g_{2}}=\frac{\rho_{1} R_{1}}{\rho_{2} R_{2}}=\frac{3}{2} \times \frac{2}{3}=1
\end{aligned}
$$

(c)

The maximum velocity with which a body must be projected in the atmosphere, so as to enable it to just overcome the gravitational pull, is known as escape velocity.
Escape velocity from earth's surface is

$$
\begin{align*}
v_{\mathrm{es}} & =\sqrt{\frac{2 G M_{e}}{R_{e}}} \\
& =\sqrt{\frac{2 G \cdot \frac{4}{3} \pi R_{e}^{3} d_{e}}{R_{e}}} \quad\left(\therefore M=\frac{4}{3} \pi R_{e}^{3} d_{e}\right) \\
v_{\mathrm{es}} & \propto \sqrt{d_{e}} \times R_{e} \ldots . . \text { (i) } \tag{i}
\end{align*}
$$

or
similarly, for a planet

$$
\begin{equation*}
v_{\mathrm{es}}^{\prime} \propto \sqrt{d_{p}} \times R_{p} \tag{ii}
\end{equation*}
$$

So, $\quad \frac{v_{\mathrm{es}}}{v_{e s}^{\prime}}=\left(\frac{d_{e}}{d_{p}}\right)^{1 / 2} \times \frac{R_{e}}{R_{p}}$
Given, $d_{p}=\frac{1}{4} d_{e}, R_{p}=2 R_{e}$

$$
\begin{aligned}
& \frac{v_{\mathrm{es}}}{v_{\mathrm{es}}}=\left(\frac{d_{e}}{d_{e}}\right. \\
&)^{\frac{1}{21}} \times \frac{R_{e}}{2 R_{e}} \\
&=(4)^{1 / 2} \times \frac{1}{2} \\
&=2 \times \frac{1}{2}=1
\end{aligned}
$$

So, $\quad \frac{v_{\mathrm{es}}}{v^{\prime}{ }_{\mathrm{es}}}=1: 1$
(a)

The value of acceleration due to gravity $g$ at height h above the surface of earth is

$$
g_{\mathrm{h}}=\frac{g}{\left(1+\frac{\mathrm{h}}{R}\right)^{2}}
$$

Where $R$ is radius of earth.
$\therefore \quad \frac{g}{g_{\mathrm{h}}}=\left(1+\frac{\mathrm{h}}{R}\right)^{2}$
(b)
$v=\sqrt{\frac{G M}{r}}$
(b)

Angular momentum is conserved in central field
(d)

The true weight of a body is given by $m g$ and with height $g$ decrease
So, $\frac{W_{s}}{W_{E}}=\frac{m \mathrm{~g}^{\prime}}{m \mathrm{~g}}=\frac{1}{[1+(\mathrm{h} / R)]^{2}}\left[\right.$ as $\left.\mathrm{g}^{\prime}=\frac{\mathrm{g}}{[1+(\mathrm{h} / R)]^{2}}\right]$
But here, $\mathrm{h}=7 R-R=6 R, i e, h / R=6$
So, $W_{S}=\frac{W_{E}}{(1+6)^{2}}=\frac{10}{49}=0.2 \mathrm{~N}$
(d)
$v_{e}=\sqrt{\frac{2 G M}{(R+h)}}$
(a)
$g^{\prime}=g\left(\frac{R}{R+h}\right)^{2}=g\left(\frac{R}{R+2 R}\right)^{2}=\frac{g}{9}$
(c)
$\frac{\mathrm{g}_{m}}{\mathrm{~g}_{e}}=\frac{G(M / 8)}{G M / R_{e}^{2}}=\frac{R_{e}^{2}}{8 R_{m}^{2}} ; \ldots(\mathrm{i})$
Given, $\frac{m g_{m}}{m g_{e}}=\frac{1}{6}$
or $\frac{\mathrm{g}_{m}}{\mathrm{~g}_{e}}=\frac{1}{6}$
From Eqs. (i) and (ii); $\frac{R_{e}^{2}}{8 R_{m}^{2}}=\frac{1}{6}$
or $R_{e}=\sqrt{8 / 6} R_{m}$

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |  |  |
| A. | A | D | D | A | D | B | D | C | B | A |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |  |  |  |
| A. | C | A | C | A | B | B | D | D | A | C |  |  |  |  |  |
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