CLASS : XIth
Solutions

## Topic :- GRAVITATION

(d)
$F \propto \frac{1}{r^{2}}$. If $r$ becomes double then $F$ reduces to $\frac{F}{4}$
(b)

We know that $g=\frac{G M}{R^{2}}$
On the planet $g_{p}=\frac{G M / 7}{R^{2} / 4}=\frac{4}{7} g$
Hence weight on the planet $=700 \times \frac{4}{7}=400 \mathrm{gm} \mathrm{wt}$
(c)

According to Kepler's third law

$$
\begin{array}{ll} 
& T^{2} \propto R^{3} \\
\Rightarrow & \frac{T_{2}}{T_{1}}=\left(\frac{R_{2}}{R_{1}}\right)^{3 / 2} \\
\therefore & \frac{T_{2}}{T_{1}}=\left(\frac{3 R}{R}\right)^{3 / 2} \\
\Rightarrow & \frac{T_{2}}{T_{1}}=\sqrt{27} \\
\therefore & T_{2}=\sqrt{27} T_{1}=\sqrt{27} \times 4=4 \sqrt{27} \mathrm{~h}
\end{array}
$$

(b)

First we have to find a point where the resultant field due to both is zero. Let the point $P$ be at a distance $x$ from centre of bigger star.
$\Rightarrow \frac{G(16 M)}{x^{2}}=\frac{G M}{(10 a-x)^{2}} \Rightarrow x=8 a\left(\right.$ from $\left.O_{1}\right)$

$i e$, once the body reaches $P$, the gravitational pull of attraction due to $M$ takes the lead to make $m$ move towards it automatically as the gravitational pull of attraction due to

16 Mvanishes ie, a minimum KE or velocity has to be imparted to $m$ from surface of 16 $M$ such that it is just able to overcome the gravitational pull of $16 M$. By law of conservation of energy.
$($ Total mechanical energy at $A)=($ Total mechanical energy at $P)$
$\Rightarrow \frac{1}{2} m v_{\text {min }}^{2}+\left[\frac{G(16 M) m}{2 a}-\frac{G M m}{8 a}\right]$
$=0+\left[\frac{G M m}{2 a}-\frac{G(16 M) m}{8 a}\right]$
$\Rightarrow \frac{1}{2} m v_{\min }^{2}=\frac{G M m}{8 a}(45) \Rightarrow v_{\min }=\frac{3}{2} \sqrt{\frac{5 G M}{a}}$

8
(a)

The net force acting on a unit mass placed at $O$ due to three equal masses $M$ at verities $A, B$ and $C$ is the gravitational field intensity at point $O$. The gravitational force on the particle placed at the point of intersection of three medians.


Since, the resultant of $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$ is equal and opposite to $\mathbf{F}_{3}$.
(b)

By the law of conservation of energy

$$
\begin{array}{rlrl} 
& (U+K)_{\text {surface }} & =(U+K)_{\infty} \\
\Rightarrow \quad-\frac{G M m}{R}+\frac{1}{2} m\left(3 v_{e}\right)^{2} & =0+\frac{1}{2} m v^{2} \\
\Rightarrow \quad-\frac{G M}{R}+\frac{9 v_{e}^{2}}{2} & =\frac{1}{2} v^{2} \\
\Rightarrow \quad v_{e}^{2} & =\frac{2 G M}{R}
\end{array}
$$

$$
\begin{aligned}
\therefore-\frac{v_{e}^{2}}{2}+\frac{9 v_{e}^{2}}{2}=\frac{1}{2} v^{2} & \Rightarrow v^{2}=8 v_{e}^{2} \\
v & =2 \sqrt{2} v_{e} \\
& =2 \sqrt{2} \times 11.2=31.7 \mathrm{kms}^{-1}
\end{aligned}
$$

(b)
$6 R$ from the surface of earth and 7R from the centre
(b)
$\frac{d A}{d t}=\frac{L}{2 m}=$ constant
(b)
$F(r)=\left\{\begin{array}{l}\frac{\sigma M m}{r^{2}} \\ \frac{\pi \sigma^{m} m}{3}\end{array}, r<R(\right.$ where $\rho$ is density of sphere)
(b)

Weight of body on the surface of earth $m g=12.6 \mathrm{~N}$
At height $h$, the value of $g^{\prime}$ is given by

$$
g^{\prime}=g \frac{R^{2}}{(R+h)^{2}}
$$

Now, $\quad h=\frac{R}{2}$
$\therefore \quad g^{\prime}=g\left(\frac{R}{R+(R / 2)}\right)^{2}=g \frac{4}{9}$
Weight at height $\mathrm{h}=m g \frac{4}{9}$

$$
=12.6 \times \frac{4}{9}=5.6 \mathrm{~N}
$$

(d)
$F=\left\{\frac{G M m}{r^{2}}\right\}, r \geq R$

## (c)

Here the force of attraction between them provides the necessary centripetal force

$\therefore \frac{m v^{2}}{R}=\frac{G m^{2}}{(4 R)^{2}}$
$\therefore v=\sqrt{\frac{G m}{4 R}}$
(b)

If a body is projected from the surface of earth with a velocity $v$ and reaches a height h , applying conservation of energy (relative to surface of earth)
$\frac{1}{2} m v^{2}=\frac{m \mathrm{gh}}{[1+(h / R)]}$
$\mathrm{h}=R=6400 \mathrm{~km}, \mathrm{~g}=10 \mathrm{~ms}^{-2}$
So, $v^{2}=\mathrm{gh}$ ie, $v=\sqrt{10 \times 6400 \times 10^{3}}=8 \mathrm{kms}^{-1}$
(b)

The value of g at latitude $\lambda$ is; $\mathrm{g}^{\prime}=\mathrm{g}-R \omega^{\prime 2} \cos ^{2} \lambda$. If earth stops rotating, $\omega=0 ; \mathrm{g}^{\prime}=\mathrm{g}$. It means the weight of body will increase
(b)

Gravitational force $\left(=\frac{G M m}{R^{3 / 2}}\right)$ provides the necessary
centripetal force (ie,mR $\omega^{2}$ )
So, $\frac{G M m}{R^{3 / 2}}=m R \omega^{2}=m R\left(\frac{2 \pi}{T}\right)^{2}=\frac{4 \pi^{2} m R}{T^{2}}$
or $\quad T^{2}=\frac{4 \pi^{2} R^{5 / 4}}{G M} i e, T^{2} \propto R^{5 / 2}$
(c)
$\frac{d A}{d t}=\frac{L}{2 m} \Rightarrow \frac{d A}{d t} \propto v r \propto \omega r^{2}$
(d)

Gravitational potential energy of body will be

At

$$
E=\frac{G M_{e} m}{r}
$$

At

$$
\begin{aligned}
r & =2 R, \\
E_{1} & =-\frac{G M_{e} m}{(2 R)} \\
r & =3 R \\
E_{2} & =-\frac{G M_{e} m}{(3 R)}
\end{aligned}
$$

Energy required to move a body of mass $m$ from on orbit of radius $2 R$ to $3 R$ is

$$
\Delta E=\frac{G M_{e} m}{R}\left[\frac{1}{2}-\frac{1}{3}\right]=\frac{G M_{e} m}{6 R}
$$

(a)
$v=\sqrt{\frac{G M}{R+h}}=\frac{1}{2} \sqrt{\frac{2 G M}{R}}$
$\Rightarrow 4 R=2(R+h) \Rightarrow h=R=6400 \mathrm{~km}$
(b)

Here, $m_{1}=m_{2}=100 \mathrm{~kg} ; r=100 \mathrm{~m}$
Acceleration of first astronaut,
$a_{1}=\frac{G m_{1} m_{2}}{r^{2}} \times \frac{1}{m_{1}}=\frac{G m_{1}}{r^{2}}$
Acceleration of second astronaut,
$a_{2}=\frac{G m_{1} m_{2}}{r^{2}} \times \frac{1}{m_{2}}=\frac{G m_{2}}{r^{2}}$
Net acceleration of approach
$a=a_{1}+a_{2}=\frac{G m_{2}}{r^{2}}+\frac{G m_{1}}{r^{2}}=\frac{2 G m_{1}}{r^{2}}$
$=\frac{2 \times\left(6.67 \times 10^{-11}\right) \times 100}{(100)^{2}}$
$=2 \times 6.67 \times 10^{-13} \mathrm{~ms}^{-2}$
As $s=\frac{1}{2} a t^{2}$
$\therefore t=\left(\frac{2 s}{a}\right)^{1 / 2}=\left[\frac{2 \times(1 / 100)}{2 \times 6.67 \times 10^{-13}}\right]^{1 / 2}$ second
On solving we get $t=1.41$ days

| ANSWER-KEY |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A. | D | B | C | B | A | B | B | B | B | B |
|  |  |  |  |  |  |  |  |  |  |  |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| A. | C | D | C | B | B | B | C | D | A | B |
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