DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :9

Topic :-TRIGONOMETRIC FUNCTIONS

861 **(c)**

$\text{cosec } 15° + \sec 15° = \dfrac{2(\sin 15° + \cos 15°)}{2 \sin 15° \cos 15°}$

$= 2\left[\dfrac{\sqrt{3}}{2\sqrt{2}} - \dfrac{1}{2\sqrt{2}} + \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}\right] / \ \sin 30°$

$= \dfrac{4\sqrt{3}}{\sqrt{2}} = 2\sqrt{6}$

862 **(d)**

We have, $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

Now, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= \sqrt{1 - \dfrac{16}{25}}\left(-\dfrac{12}{13}\right) - \dfrac{4}{5}\sqrt{1 - \dfrac{144}{169}}$

$= -\dfrac{3}{5} \times \dfrac{12}{13} - \dfrac{4}{5}\left(-\dfrac{5}{13}\right)$

$= -\dfrac{36}{65} + \dfrac{20}{65} = -\dfrac{16}{65}$

863 **(a)**

Given that, $\frac{1}{\tan\theta} + \tan\theta = m$

$\Rightarrow 1 + \tan^2\theta = m \tan\theta$

$\Rightarrow \sec^2\theta = m\tan\theta$ ...(i)

and $\sec\theta - \cos\theta = n$

$\Rightarrow \sec^2\theta - 1 = n\sec\theta$

$\Rightarrow \tan^2\theta = n\sec\theta$

$\Rightarrow \tan^4\theta = n^2\sec^2\theta = n^2 \cdot m\tan\theta$ [from Eq.(i)]

$\Rightarrow \tan^3\theta = n^2 m$ $(\because \tan\theta \neq 0)$

$\Rightarrow \tan\theta = (n^2 m)^{1/3}$ ...(ii)

From Eq. (i), we get

$\sec^2\theta = m\,(n^2m)^{1/3}$

As we know that, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow m(mn^2)^{1/3} - \,(n^2m)^{2/3} = 1$

$\Rightarrow m(mn^2)^{1/3} - \,n(nm^2)^{1/3} = 1$

864 **(c)**
We have,
$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3\sin A\sin B$
$\Rightarrow (\sin A + \sin B)^2 - \sin^2 C = 3\sin A\sin B$
$\Rightarrow \sin^2 A + \sin^2 B - \sin^2 C = \sin A\sin B$
$\Rightarrow \sin^2 A + \sin(B+C)\sin(B-C) = \sin A\sin B$
$\Rightarrow \sin^2 A + \sin A\sin(B-C) = \sin A\sin B$
$\Rightarrow \sin A[\sin(B+C) + \sin(B-C)] = \sin A\sin B$
$\Rightarrow 2\sin A\sin B\cos C = \sin A\sin B$
$\Rightarrow \cos C = 1/2 \qquad [\because \sin A\sin B \neq 0]$
$\Rightarrow C = 60°$
865 **(b)**
Given, $\cos 2x + 7 = a\,(2 - \sin x)$
$\Rightarrow\; 1 - 2\sin^2 x + 7 = 2a - a\sin x$
$\Rightarrow 2\sin^2 x - a\sin x + (2a - 8) = 0$
$\therefore\; \sin x = \dfrac{a \pm \sqrt{(-a)^2 - 8(2a-8)}}{2\times 2}$
$= \dfrac{a \pm (a-8)}{4}$
For (+) sign,
$\sin x = \dfrac{a-4}{2}$
For (−) sign,
$\sin x = 2$ which is not possible
We know $-1 \le \sin x \le 1$
$\therefore\; -1 \le \dfrac{a-4}{2} \le 1 \;\Rightarrow\; 2 \le a \le 6$
866 **(c)**

$\cos^2 B + \cos^2 C = \cos^2 B + \cos^2\left(\dfrac{\pi}{2} - B\right)$
$= \cos^2 B + \sin^2 B = 1$
867 **(d)**
We have,

$b^2 \sin 2C + c^2 \sin 2B$
$= b^2 \cdot (2 \sin C \cos C) + c^2 \cdot (2 \sin B \cos B)$
$= 2(b \sin C)(b \cos C) + 2(c \sin B)(c \cos B)$
$= 2(c \sin B)(b \cos c) + 2(c \sin B)(c \cos B)$
$\left[\because \frac{b}{\sin B} = \frac{c}{\sin C}\right]$

$= 2\,c \sin B(b \cos C + c \cos B) = 2\,ac \sin B = 4\,\Delta$

868 **(a)**

Since the angles of ΔABC are in A.P.
$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180° \Rightarrow B = 60°$

Now, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \sin A = \frac{a}{b} \sin B = \frac{24}{22} \sin 60° = \frac{6\sqrt{3}}{11}$

$\Rightarrow \cos A = \frac{\sqrt{13}}{11}$

We have,
$\sin C = \sin\{180° - (A + B)\}$
$\Rightarrow \sin C = \sin(A + B)$

$\Rightarrow \sin C = \sin A \cos B + \cos A \sin B$

$\Rightarrow \sin C = \left(\frac{6\sqrt{3}}{11}\right)\left(\frac{1}{2}\right) + \frac{\sqrt{13}}{11}\left(\frac{\sqrt{3}}{2}\right) = \frac{6\sqrt{3} + \sqrt{39}}{22}$

$\therefore c = \frac{b \sin C}{\sin B} \Rightarrow c = 12 + 2\sqrt{13}$

869 **(a)**

We have, $\angle BFC = \frac{\pi}{2} = \angle BEC$

So, the circle with BC as diameter will pass through E and F. Clearly, the circle with BC as diameter is the circumcircle of ΔBEF such that $\angle FBE = 90° - A$

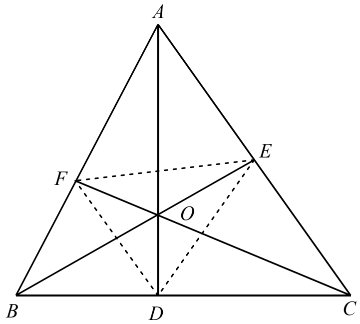
$\therefore FE = 2\left(\frac{a}{2}\right) \sin \angle FBE$ [Using : $a = 2R \sin A$]

$\Rightarrow FE = a \cos A$

Let R_1 be the radius of the circumcircle of ΔDEF. Then,

$R_1 = \frac{FE}{2 \sin \angle FDE} = \frac{a \cos A}{2 \sin(180° - 2A)}$

$\Rightarrow R_1 = \frac{a \cos A}{4 \sin A \cos A} = \frac{a}{4 \sin A} = \frac{R}{2}$



870 **(b)**

Clearly, the equation $x^2+\sqrt{2}x+1=0$ has imaginary roots. So, the two equations have both common roots

$\therefore \frac{a}{1}=\frac{b}{\sqrt{2}}=\frac{c}{1}$

$\Rightarrow \frac{\sin A}{1}=\frac{\sin B}{\sqrt{2}}=\frac{\sin C}{1}$

$\Rightarrow \frac{\sin A}{1/\sqrt{2}}=\frac{\sin B}{1}=\frac{\sin C}{1/\sqrt{2}}$

$\Rightarrow A=\frac{\pi}{4}, B=\frac{\pi}{2}, C=\frac{\pi}{4}$

871 **(a)**

We have,

$s=\frac{3a}{2}$ and $\Delta=\frac{\sqrt{3}}{4}a^2 \therefore r=\frac{\Delta}{s}=\frac{a}{2\sqrt{3}}$

Let the length of each side of the square inscribed in the incircle be x. Then,

$x^2+x^2=(\text{Diameter})^2$

$\Rightarrow 2x^2=\frac{a^2}{3} \Rightarrow x^2=\frac{a^2}{6} \Rightarrow$ Area of the square $=\frac{a^2}{6}$

872 **(a)**

$\cos 1°.\cos 2° \ldots\ldots \cos 179°$

$=\cos 1°.\cos 2° \ldots\ldots \cos 90°.\cos 179°$

$=0 \quad [\because \cos 90°=0]$

873 **(a)**

Given equation is

$2^{\cos 2x}+1=3.2^{-\sin x}$

By taking option (a)

Let $x=n\pi$

When, $n=1,\ x=\pi$

$\therefore \quad 2^{\cos 2\pi}+1=3.2^{-\sin \pi}$

$\Rightarrow\ 2+1=3.2° \ \Rightarrow\ 3=3$

When $n=2, x=2\pi$

$\therefore\ 2^{\cos 4\pi}+1=3.2^{-\sin 2\pi}$

$\Rightarrow\ 2^1+1=3.2°$

$\Rightarrow\ 3=3$

874 **(b)**

On squaring given equation, we get

$\sin^2 A + 6\cos^2 A - 2\sqrt{6}\sin A\cos A = 7\cos^2 A$

$\Rightarrow \sin^2 A + 6(1-\sin^2 A) = \cos^2 A + 6\cos^2 A + 2\sqrt{6}\sin A\cos A$

$\Rightarrow \sin^2 A - 6\cos^2 A + 6 = \cos^2 A + 6\sin^2 A + 2\sqrt{6}\sin A\cos A$

$\Rightarrow 7\sin^2 A = (\cos A + \sqrt{6}\sin A)^2$

$\Rightarrow \pm\sqrt{7}\sin A = \cos A + \sqrt{6}\sin A$

Alternate

Given, $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$

Replacing A by $90° + A$, we get

$\sin(90° + A) - \sqrt{6}\cos(90° + A)$

$= \sqrt{7}\cos(90° + A)$

$\Rightarrow \cos A + \sqrt{6}\sin A = -\sqrt{7}\sin A$

875 **(b)**

We have,

$y = \frac{\tan x}{\tan 3x}$

$\Rightarrow y = \frac{1 - 3\tan^2 x}{3 - \tan^2 x}$

$\Rightarrow 3y - y\tan^2 x = 1 - 3\tan^2 x$

$\Rightarrow \tan^2 x(y-3) = 1 - 3y$

$\Rightarrow \tan^2 x = \frac{y-3}{1-3y}$

$\Rightarrow -\frac{y-3}{3y-1} \geq 0 \ [\because \tan^2 x \geq 0]$

$\Rightarrow \frac{y-3}{3y-1} \leq 0 \Rightarrow \frac{1}{3} \leq y < 3 \Rightarrow y \in [1/3, 3]$

876 **(c)**

We have, $\sqrt{\text{cosec}^2\alpha + 2\cot\alpha}$

$= \sqrt{1 + \cot^2\alpha + 2\cot\alpha} = |1 + \cot\alpha|$

But $\frac{3\pi}{4} < \alpha < \pi$

$\Rightarrow \cot\alpha < -1 \Rightarrow 1 + \cot\alpha < 0$

Hence, $|1 + \cot\alpha| = -(1 + \cot\alpha)$

877 **(c)**

Since $-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$. Therefore, $a\sin x + b\cos x = c$ has no solution for $|c| > \sqrt{a^2+b^2}$

878 **(c)**

We have,

$\tan\theta + \sec\theta = 2\cos\theta$

$\Rightarrow 1 + \sin\theta = 2\cos^2\theta$

$\Rightarrow 1 + \sin\theta = 2 - 2\sin^2\theta$

$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, \sin\theta = -1$

$\Rightarrow \sin\theta = \frac{1}{2} \quad \left[\begin{matrix} \because \sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2} \\ \text{for which the equation is not defined} \end{matrix}\right]$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6} \qquad [\because \theta \in [0, 2\pi]]$

Hence, the given equation has two solutions in $[0, 2\pi]$

879 **(c)**

Given, $\sin\pi(x^2 + x) - \sin\pi x^2 = 0$

$\Rightarrow 2\cos\pi\left(\frac{2x^2 + x}{2}\right)\sin\frac{\pi x}{2} = 0$

$\Rightarrow \pi\left(\frac{2x^2 + x}{2}\right) = n\pi + \frac{\pi}{2}$

$\Rightarrow 2x^2 + x = 2n + 1$

$\Rightarrow 2x^2 + x - p' = 0$, where $p' = 2n + 1$, is an odd integer

$\therefore x = \frac{-1 \pm \sqrt{1 + 8p'}}{4}$ [put $1 + 8p' = p$]

$\therefore x = \frac{-1 \pm \sqrt{p}}{4} \Rightarrow x = \frac{\sqrt{p} - 1}{4} \quad \left[\text{neglect } x = \frac{-1 - \sqrt{p}}{4}\right]$

880 **(b)**

Given that, $3\sin^2 A + 2\sin^2 B = 1$ ...(i)

and $3\sin 2A - 2\sin 2B = 0$ ...(ii)

From Eq. (i)

$3\left(\frac{1 - \cos 2A}{2}\right) + 2\left(\frac{1 - \cos 2B}{2}\right) = 1$

$\Rightarrow 3\cos 2A + 2\cos 2B = 3$ ...(iii)

$\Rightarrow 3\cos 2A = 3 - 2\cos 2B$

$\Rightarrow 9\cos^2 2A = 9 + 4\cos^2 2B - 12\cos 2B$

$\Rightarrow 9(1 - \sin^2 2A) = 9 + 4\cos^2 2B - 12\cos 2B$

$\Rightarrow 9 - 4\sin^2 2B = 9 + 4\cos^2 2B - 12\cos 2B$ [from Eq. (ii)]

$\Rightarrow -4(1 - \cos^2 2B) = 4\cos^2 2B - 12\cos 2B$

$\Rightarrow -4 = -12\cos 2B$

$\Rightarrow \cos 2B = \frac{1}{3}$

Now, from Eq. (iii)

$$\cos 2A = \frac{7}{9} \Rightarrow 2\cos^2 A - 1 = \frac{7}{9}$$

$$\Rightarrow \cos A = \frac{2\sqrt{2}}{3}$$

$$\therefore A + 2B = \cos^{-1}\frac{2\sqrt{2}}{3} + \cos^{-1}\frac{1}{3}$$

$$= \cos^{-1}\left(\frac{2\sqrt{2}}{3}\cdot\frac{1}{3} - \sqrt{1-\frac{8}{9}}\sqrt{1-\frac{1}{9}}\right)$$

$$= \cos^{-1}\left(\frac{2\sqrt{2}}{9} - \frac{2\sqrt{2}}{9}\right)$$

$$= \cos^{-1}(0) = \frac{\pi}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	A	C	B	C	D	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	B	B	C	C	C	C	B