DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Subject : MATHS
Date :
DPP No. :8

Solutions

Topic :-TRIGONOMETRIC FUNCTIONS

84 **(a)**

2 Let ABC be the triangle such that $a = 2\sqrt{2}$ cm, $b = 2\sqrt{3}$ cm and $\angle A = \frac{\pi}{4}$

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{12 + c^2 - 8}{4\sqrt{3}\,c}$

$\Rightarrow 4 + c^2 = 2\sqrt{6}\,c$

$\Rightarrow c^2 - 2\sqrt{6}\,c + 4 = 0$

$\Rightarrow c = \frac{2\sqrt{6} \pm \sqrt{24 - 16}}{2}$

$\Rightarrow c = \sqrt{6} \pm \sqrt{2} \Rightarrow c = \sqrt{6} + \sqrt{2}$ [$\because c$ is the largest side]

84 **(b)**

3 We have,

$r\sin\theta = 3$ and $r = 4(1 + \sin\theta)$

$\Rightarrow r = 4 + \frac{12}{r}$

$\Rightarrow r^2 - 4r - 12 = 0$

$\Rightarrow (r - 6)(r + 2) = 0$

$\Rightarrow r = 6$ [$\because r > 0$]

$\therefore r\sin\theta = 3 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, the total number of ordered pairs of the form (r, θ) is $1 \times 2 = 2$

84 **(d)**

4 We have,

$\sin 65° + \sin 43° - \sin 29° - \sin 7°$

$= (\sin 65° + \sin 43°) - (\sin 29° + \sin 7°)$

$= 2\sin 54° \cos 11° - 2\sin 18° \cos 11°$

$= 2\cos 11° (\cos 36° - \sin 18°)$

$= 2\cos 11° \left(\frac{\sqrt{5} + 1}{4} - \frac{\sqrt{5} - 1}{4}\right) = \cos 11°$

84 **(c)**

5 We have,

$\sin B = \frac{1}{5}\sin(2A + B)$

$\Rightarrow \frac{\sin(2A + B)}{\sin B} = \frac{5}{1}$

$\Rightarrow \frac{\sin(2A + B) + \sin B}{\sin(2A + B) - \sin B} = \frac{5 + 1}{5 - 1}$

$\Rightarrow \frac{2\sin(A + B)\cos A}{2\sin A\cos(A + B)} = \frac{3}{2}$

$\Rightarrow \frac{\tan(A + B)}{\tan A} = \frac{3}{2}$

84 **(b)**

6 Since, $A + B + C = \pi$

$\Rightarrow a = \pi - (B + C)$

We have, $\cos A = \cos B \cos C$

$\Rightarrow \cos[\pi - (B + C)] = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow -[\cos B \cos C - \sin B \sin C] = \cos B \cos C$

$\Rightarrow \sin B \sin C = 2\cos B \cos C$

$\Rightarrow \tan B \tan C = 2$

84 **(a)**

7 We have,

$\sin x + \operatorname{cosec} x = 2 \Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$

$\therefore \sin^n x + \operatorname{cosec}^n x = 1 + 1 = 2$

84 **(a)**

8 We have,

$\frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin(A - B)}{\sin(A + B)}$

$\Rightarrow \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B} = \frac{\sin^2 A - \sin^2 B}{\sin^2(A + B)}$

$\Rightarrow (\sin^2 A - \sin^2 B)(\sin^2 A + \sin^2 B - \sin^2 C) = 0$

$\Rightarrow \sin(A + B)\sin(A - B)(\sin^2 A + \sin^2 B - \sin^2 C)$

$\Rightarrow \sin(A - B) = 0$ or, $\sin^2 A + \sin^2 B = \sin^2 C$

$\Rightarrow A = B$ or, $a^2 + b^2 = c^2$

$\Rightarrow \Delta ABC$ is either right angled or isosceles

84 **(c)**

9 We have,

$\cos A = \cos B \cos C$

$\Rightarrow \cos\{\pi - (B + C)\} = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow 2\cos B \cos C = \sin B \sin C$

$\Rightarrow \cot B \cot C = \frac{1}{2}$

85 **(c)**

0 We have,

$2s = a + b + c = 13 + 14 + 15$

$\Rightarrow s = 21$

$\Rightarrow s - a = 8, s - b = 7$ and $s - c = 6$

Now,

$$\frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3} = \frac{s-a}{\Delta} : \frac{s-b}{\Delta} : \frac{s-c}{\Delta}$$

$$\Rightarrow \frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3} = s - a : s - b : s - c = 8 : 7 : 6$$

851 **(a)**

We have,

$$\frac{\sin 7\theta + 6 \sin 5\theta + 17 \sin 3\theta + 12 \sin \theta}{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}$$

$$= \frac{(\sin 7\theta + \sin 5\theta) + 5(\sin 5\theta + \sin 3\theta) + 12(\sin 3\theta + \sin \theta)}{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}$$

$$= \frac{2 \sin 6\theta \cos \theta + 10 \sin 4\theta \cos \theta + 24 \sin 2\theta \cos \theta}{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}$$

$$= \frac{2 \cos \theta (\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta)}{\sin 6\theta + 5 \sin 4\theta + 12 \sin 2\theta}$$

$= 2 \cos \theta$

852 **(a)**
Let the angles be $A = x - d, B = x, C = x + d$. Then,
$x - d + x + x + d = 180° \Rightarrow x = 60°$
Therefore, two larger angles are $B = 60°$ and C
Hence, $b = 9$ and $c = 10$
Now,
$$\cos B = \frac{c^2 + a^2 - b^2}{2\,ac}$$
$$\Rightarrow \frac{1}{2} = \frac{100 + a^2 - 81}{20\,a} \Rightarrow a^2 - 10\,a + 19 = 0 \Rightarrow a = 5 \pm \sqrt{6}$$
853 **(b)**
Since, $\cos 2x, \frac{1}{2}, \sin 2x$ are in AP
$\Rightarrow \cos 2x + \sin 2x = 1$
$\Rightarrow \sin 2x = 1 - \cos 2x = 2\sin^2 x$
$\Rightarrow 2\sin x(\cos x - \sin x) = 0$
$\Rightarrow \sin x = 0$ or $\cos x - \sin x = 0$
$\Rightarrow x = n\pi$ or $\tan x = 1$
$\Rightarrow x = n\pi$ or $x = n\pi + \frac{\pi}{4}$
Thus, required values of x are $n\pi$ and $n\pi + \frac{\pi}{4}$
854 **(b)**
$$\cos\frac{\pi}{18} + \cos\frac{2\pi}{18} + \dots + \cos\frac{16\pi}{18} + \cos\frac{17\pi}{18} + \cos\pi$$
$$= \cos\frac{\pi}{18} + \cos\frac{2\pi}{18} + \dots - \cos\frac{2\pi}{18} - \cos\frac{\pi}{18} + \cos\pi$$
$= \cos\pi = -1$
855 **(b)**
Given that, $\sin\theta + \cos\theta = 1$
$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$
$$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$$
$$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$$
856 **(d)**
We have,
$0 < x < \pi \Rightarrow \sin x > 0$
Now,
$1 + \sin x + \sin^2 x + \dots\infty = 4 + 2\sqrt{3}$
$$\Rightarrow \frac{1}{1 - \sin x} = 4 + 2\sqrt{3}$$

$\Rightarrow \sin x = 1 - \frac{1}{4+2\sqrt{3}}$

$\Rightarrow \sin x = \frac{3+2\sqrt{3}}{4+2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$ or, $\frac{2\pi}{3}$

857 **(b)**

$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{a}{b}$

$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{a}{b}$

$\Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$

858 **(b)**

The given equation can be written as

$\cos(\pi\tan\theta) = \cos\left(\frac{\pi}{2} - \pi\cot\theta\right)$

$\Rightarrow \pi\tan\theta = 2n\pi \pm \left(\frac{\pi}{2} - \pi\cot\theta\right), n \in Z$

$\Rightarrow \tan\theta = 2n \pm \left(\frac{1}{2} - \cot\theta\right), n \in Z$

$\Rightarrow \tan\theta - \cot\theta = 2n - \frac{1}{2}, n \in Z$ [Taking negative sign]

$\Rightarrow \frac{\tan^2\theta - 1}{\tan\theta} = 2n - \frac{1}{2}$

$\Rightarrow \frac{\tan^2\theta - 1}{2\tan\theta} = n - \frac{1}{4}$

$\Rightarrow \frac{1-\tan^2\theta}{2\tan\theta} = -n + \frac{1}{4}$

$\Rightarrow \cot 2\theta = m + \frac{1}{4}$, where $m = -n \in Z$

859 **(c)**

From Questions 47, we have

$\Delta = \frac{1}{2}ap_1, \Delta = \frac{1}{2}bp_2, \Delta = \frac{1}{2}cp_3$

Now,

p_1, p_2, p_3 are in A.P.

$\Rightarrow \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in A.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $\Rightarrow a, b, c$ are in H.P.

860 **(d)**

$$\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}=\frac{\sin 2^4\frac{\pi}{5}}{2^4\sin\frac{\pi}{5}}\left(\because \cos x\cos 2x\cos 4x\ldots\cos 2nx=\frac{\sin 2^n x}{2^n\sin x}\right)$$

$$=\frac{\sin\frac{16\pi}{5}}{16\sin\frac{\pi}{5}}=\frac{\sin\left(3\pi+\frac{\pi}{5}\right)}{16\sin\frac{\pi}{5}}$$

$$=\frac{-\sin\frac{\pi}{5}}{16\sin\frac{\pi}{5}}=-\frac{1}{16}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	D	C	B	A	A	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	B	B	B	D	B	B	C	D