DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :7

Topic :-TRIGONOMETRIC FUNCTIONS

821 **(a)**

Given, $\sin 2x + \cos 4x = 2$

$\Rightarrow \sin 2x + 1 - 2\sin^2 2x = 2$

$\Rightarrow 2\sin^2 2x - \sin 2x + 1 = 0$

Now, Discriminant, $D = (-1)^2 - 4.2.1 = -7 < 0$

Hence, it is an imaginary equation, so the real root does not exist.

822 **(d)**

We have,

$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1 \quad [\because -1 \le \sin x \le 1]$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \dfrac{\pi}{2} \Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

823 **(b)**

We have,

$k\sin x + (1 - 2\sin^2 x) = 2k - 7$

$\Rightarrow 2\sin^2 x - k\sin x + 2(k-4) = 0$

$\Rightarrow \sin x = \dfrac{k \pm \sqrt{k^2 - 16k + 64}}{4} = \dfrac{k \pm (k-8)}{4} = \dfrac{1}{2}(k-4), 2$

$\Rightarrow \sin x = \dfrac{1}{2}(k-4) \qquad [\because \sin x \ne 2]$

Now, $-1 \le \sin x \le 1 \Rightarrow -1 \le \frac{k-4}{2} \le 1 \Rightarrow 2 \le k \le 6$

824 **(b)**

Given that, $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$

$= \dfrac{\cos A\cos C - \sin A\sin C}{\cos A\cos C + \sin A\sin C}$

$\Rightarrow \dfrac{1 - \tan^2 B}{1 + \tan^2 B} = \dfrac{1 - \tan A\tan C}{1 + \tan A\tan C}$

$\Rightarrow 1 + \tan^2 B - \tan A\tan C - \tan A\tan C\tan^2 B$

$= 1 - \tan^2 B + \tan A \tan C - \tan A \tan C \tan^2 B$

$\Rightarrow 2\tan^2 B = 2\tan A \tan C$

$\Rightarrow \tan^2 B = \tan A \tan C$

Hence, $\tan A$, $\tan B$ and $\tan C$ will be in GP

825 **(c)**

We have,

$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$= \left(\cot \frac{A-B}{2}\right)^n + \left(-\cot \frac{A-B}{2}\right)^n$

$= \{1 + (-1)^n\} \cot^n \left(\frac{A-B}{2}\right)$

$= 0 \times \cot^n \left(\frac{A-B}{2}\right) = 0 \quad [\because n \text{ is odd}]$

826 **(a)**

We have,

$a \tan\theta + b \sec\theta = c$

$\Rightarrow b \sec\theta = c - a\tan\theta$

$\Rightarrow b^2 \sec^2\theta = c^2 + a^2 \tan^2\theta - 2\,ac \tan\theta$

$\Rightarrow b^2(1 + \tan^2\theta) = c^2 + a^2\tan^2\theta - 2\,ac\tan\theta$

$\Rightarrow \tan^2\theta(b^2 - a^2) + 2\,ac\tan\theta + b^2 - c^2 = 0$

Since $\tan\alpha$ and $\tan\beta$ are roots of this equation

$\therefore \tan\alpha + \tan\beta = \frac{-2\,ac}{b^2 - a^2}$ and $\tan\alpha\tan\beta = \frac{b^2 - c^2}{a^2 - c^2}$

Now,

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-\frac{2\,ac}{b^2 - a^2}}{1 - \frac{b^2 - c^2}{b^2 - a^2}} = \frac{2\,ac}{a^2 - c^2}$$

827 **(b)**

$\tan\theta + \frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} + \frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta} = 3$

$\Rightarrow \tan\theta + \frac{8\tan\theta}{1 - 3\tan^2\theta} = 3$

$\Rightarrow \frac{9\tan\theta - 3\tan^3\theta}{1 - 3\tan^2\theta} = 3$

$\Rightarrow \quad 3\tan 3\theta = 3 \quad \Rightarrow \quad \tan 3\theta = 1$

828 **(a)**

Since, $y = 1 + 4\sin^2 x \cos^2 x$

$\Rightarrow y = 1 + \sin^2 2x$

We know that, $0 \leq \sin^2 2x \leq 1$

$\Rightarrow 1 \leq 1 + \sin^2 2x \leq 2$

$\Rightarrow 1 \leq y \leq 2$

829 **(a)**

$\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma$

$= \sin^2 \alpha + \sin(\beta - \gamma) \sin(\beta + \gamma)$

$= \sin^2 \alpha \sin(\pi - \alpha) \sin(\beta + \gamma) \quad [\because \alpha + \beta - \gamma = \pi]$

$= \sin \alpha [\sin \alpha + \sin(\beta + \gamma)]$

$= \sin \alpha [\sin\{\pi - (\beta - \gamma)\} + \sin(\beta + \gamma)]$

$= \sin \alpha [\sin(\beta - \gamma) + \sin(\beta + \gamma)]$

$= \sin \alpha [2 \sin \beta \cos \gamma]$

$= 2 \sin \alpha \sin \beta \cos \gamma$

830 **(a)**

We have,

$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}$

$= \frac{s(s-a)}{\Delta} + \frac{s(s-b)}{\Delta} + \frac{s(s-c)}{\Delta} = \frac{s}{\Delta}(3s - 2s) = \frac{s^2}{\Delta}$

And,

$\cot A + \cot B + \cot C$

$= \frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}$

$= \frac{b^2 + c^2 - a^2}{2\,bc \sin A} + \frac{c^2 + a^2 - b^2}{2ac \sin B} + \frac{a^2 + b^2 - c^2}{2ab \sin C}$

$= \frac{b^2 + c^2 - a^2}{4\,\Delta} + \frac{c^2 + a^2 - b^2}{4\,\Delta} + \frac{a^2 + b^2 - c^2}{4\,\Delta}$

$= \frac{a^2 + b^2 + c^2}{4\,\Delta}$

$\therefore \frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C} = \frac{\frac{s^2}{\Delta}}{\frac{a^2 + b^2 + c^2}{4\,\Delta}}$

$= \frac{(2s)^2}{a^2 + b^2 + c^2} = \frac{(a + b + c)^2}{a^2 + b^2 + c^2}$

831 **(a)**

$\because \quad (\tan \alpha - \cot \alpha)^2 \geq 0$

$\Rightarrow\ \tan^2\alpha + \cot^2\alpha - 2 \geq 0$
$\Rightarrow\ \tan^2\alpha + \cot^2\alpha \geq 2$
832 **(a)**
We have,
$\tan m\,\theta = \tan n\,\theta$
$\Rightarrow m\,\theta = t\,\pi + n\,\theta$, where $r \in Z$
$\Rightarrow \theta = \dfrac{r\,\pi}{m-n}, r \in Z$
Clearly, these values of θ from an A.P. with common difference $\frac{\pi}{m-n}$
833 **(a)**
We have,
$\dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$
$\Rightarrow \dfrac{\sin(B+C)}{\sin(a+B)} = \dfrac{\sin(A-B)}{\sin(B-C)}$
$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$
$\Rightarrow b^2 - c^2 = a^2 - b^2$
$\Rightarrow a^2, b^2, c^2$ are in A.P.
834 **(a)**
Let $\sec\theta - \tan\theta = \lambda$ ...(i)
Then,
$(\sec\theta + \tan\theta) = \dfrac{1}{\sec\theta - \tan\theta}$
$\Rightarrow \sec\theta + \tan\theta = \frac{1}{\lambda}$ ...(ii)
$\therefore 2\tan\theta = \frac{1}{\lambda} + \lambda$ [On subtracting (i) from (ii)]
$\Rightarrow 2x - \dfrac{1}{2x} = \dfrac{1}{\lambda} - \lambda$
$\Rightarrow \lambda = \dfrac{1}{2x}, -2x \Rightarrow \sec\theta - \tan\theta = \dfrac{1}{2x}, -2x$
835 **(d)**
We observe that the LHS of the given equation is not defined for $x = n\,\pi, n \in Z$
Now,
$\cot x - \operatorname{cosec} x = 2\sin x$
$\Rightarrow \cot x - 1 = 2\sin^2 x$
$\Rightarrow 2\cos^2 x + \cos x - 3 = 0$
$\Rightarrow (2\cos x + 3)(\cos x - 1) = 0$
$\Rightarrow \cos x = 1$ $[\because 2\cos x + 3 \neq 0]$
$\Rightarrow x = 0, 2\,\pi$
But, $x \neq n\,\pi, n \in Z$
Hence, the given equation has no solution
837 **(d)**

Given, $\frac{\sin x}{\sin y} = \frac{1}{2}, \frac{\cos x}{\cos y} = \frac{3}{2}$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{1}{3}$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{4 \tan x}{1 - 3 \tan^2 x}$$

Also, $\sin y = 2 \sin x, \cos y = \frac{2}{3} \cos x$

$$\Rightarrow \sin^2 y + \cos^2 y = 4 \sin^2 x + \frac{4 \cos^2 x}{9} = 1$$

$$\Rightarrow 36 \tan^2 x + 4 = 9 \sec^2 x = 9(1 + \tan^2 x)$$

$$\Rightarrow 27 \tan^2 x = 5$$

$$\Rightarrow \tan x = \frac{\sqrt{5}}{3\sqrt{3}}$$

$$\Rightarrow \tan(x + y) = \frac{\frac{4\sqrt{5}}{3\sqrt{3}}}{1 - \frac{15}{27}} = \sqrt{15}$$

840 **(d)**

$$\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{1 + \tan 9°}{1 - \tan 9°}$$

$$= \tan(45° + 9°)$$

$$= \tan 54°$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	B	B	C	A	B	A	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	A	A	D	C	D	A	A	D