DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : MATHS
DPP No. :6

Topic :-TRIGONOMETRIC FUNCTIONS

801 **(a)**

$\cos(\alpha+\beta) = -\frac{12}{13}$

Here, $0 < (\alpha+\beta) < \pi$

$\therefore \sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$

$= \sqrt{1-\frac{144}{169}}$

$= \frac{5}{13}$

Now, $\sin\beta = \sin[(\alpha+\beta)-\alpha]$

$= \sin(\alpha+\beta)\cos\alpha - \cos(\alpha+\beta)\sin\alpha$

$= \frac{5}{13}.\frac{3}{5} - \left(-\frac{12}{13}\right).\frac{4}{5}$

$= \frac{15}{65}+\frac{48}{65}$

$= \frac{63}{65}$

802 **(c)**

We have,

$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$

$= \sin\left(\frac{\pi}{2}-\frac{3\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{2\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{7}\right)\sin\frac{\pi}{2}$

$= \cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}$

$= -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -\frac{\sin(2^3\pi/7)}{2^3\sin\pi/7} = -\frac{\sin(8\pi/7)}{8\sin\pi/7} = \frac{1}{8}$

803 **(b)**

We have,

$\sin\theta\cos\alpha + \cos\theta\sin\alpha = 2\,k\sin\theta\cos\theta$

$\Rightarrow \cos\alpha \dfrac{2t}{1+t^2} + \sin\alpha \dfrac{1-t^2}{1+t^2} = 2k \dfrac{2t}{1+t^2} \times \dfrac{1-t^2}{1+t^2}$

where $t = \tan\dfrac{\theta}{2}$

$\Rightarrow \sin\alpha\, t^4 - (2\cos\alpha + 4k)t^3 + t(4k - 2\cos\alpha) - \sin\alpha = 0$

$\Rightarrow S_1 = 2\cos\alpha + 4k, S_2 = 0$

$S_3 = 2\cos\alpha - 4k, S_4 = -1$

where S_r denotes the sum of the product of roots taken r at a time

Now,

$\tan\left(\dfrac{\theta_1}{2} + \dfrac{\theta_2}{2} + \dfrac{\theta_3}{2} + \dfrac{\theta_4}{2}\right) = \dfrac{S_1 - S_3}{1 - S_2 + S_4} = \infty = \tan\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \dfrac{\theta_1}{2} + \dfrac{\theta_2}{2} + \dfrac{\theta_3}{2} + \dfrac{\theta_4}{2} = n\pi + \dfrac{\pi}{2}, n \in Z$

$\Rightarrow \theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n+1)\pi, n \in Z$

804 **(b)**

Let r be the radius of the circle. Then,

$\dfrac{3\pi}{4} = \dfrac{15\pi}{r} \Rightarrow r = 20$ cm

805 **(a)**

We know that

$\cot\alpha - \tan\alpha = 2\cot 2\alpha$

$\therefore \cot\theta - \tan\theta - 2\tan 2\theta - 4\tan 4\theta - 8\cot 8\theta$

$= 2\cot 2\theta - 2\tan 2\theta - 4\tan 4\theta - 8\cot 8\theta$

$= 2(2\cot 4\theta) - 4\tan 4\theta - 8\cot 8\theta$

$= 4\cot 4\theta - 4\tan 4\theta - 8\cot 8\theta$

$= 4(\cot 4\theta - \tan 4\theta) - 8\cot 8\theta$

$= 4 \times 2\cot 8\theta - 8\cot 8\theta = 0$

806 **(d)**

We have,

$b = \sqrt{3}, c = 1$ and $A = 30°$

$\therefore \cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{4 - a^2}{2\sqrt{3}} \Rightarrow a = 1$

$\therefore \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\Rightarrow 2 = \dfrac{\sqrt{3}}{\sin B} = \dfrac{1}{\sin C}$

$\Rightarrow \sin B = \dfrac{\sqrt{3}}{2}$ and $\sin C = \dfrac{1}{2}$

$\Rightarrow B = 120°$ and $C = 30°$ $[\because b > c \therefore B > C]$

807 **(c)**

Here, $a = 3,\ b = 4$

$\therefore$ maximum value $= \sqrt{3^2 + 4^2} = 5$

808 **(a)**

Let ABC be the triangle such that $a = 2, b = \sqrt{6}$ and $c = \sqrt{3} - 1$

Clearly, $b > a > c$

So, B is the greatest angle and C is the smallest angle

Now,

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

$$\Rightarrow \cos B = \frac{(\sqrt{3} - 1)^2 + 4 - 6}{4(\sqrt{3} - 1)^2} = -\frac{1}{2} \Rightarrow B = 120°$$

And,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos C = \frac{4 + 6 - (\sqrt{3} - 1)^2}{4\sqrt{6}} = \frac{\sqrt{3} + 1}{2\sqrt{2}} \Rightarrow C = 15°$$

809 **(b)**

We have,

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{16} + \frac{1}{48} + \frac{1}{24} \Rightarrow r = 8$$

810 **(c)**

Given, $3\sin^2 x - 7\sin x + 2 = 0$

$(3\sin x - 1)(\sin x - 2) = 0$

$$\Rightarrow \sin x = \frac{1}{3} \text{ or } 2$$

$$\Rightarrow \sin x = \frac{1}{3} \quad [\because \sin x \neq 2]$$

Let $\sin^{-1}\frac{1}{3} = \alpha$, $0 < \alpha < \frac{\pi}{2}$, then $\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$ are the solution in $[0, 5\pi]$

Hence, required number of solutions are 6

811 **(a)**

We have, $\cos^2\theta = \cos 2\theta$

$\Rightarrow \cos^2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos^2\theta = 1 \Rightarrow \theta = n\pi$

812 **(a)**

The given equation is

$3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$

$\Rightarrow 3^{\sin 2x + 2\cos^2 x} + 3^{3 - (\sin 2x + 2\cos^2 x)} = 28$

$$\Rightarrow y + \frac{27}{y} = 28, \text{ where } y = 3^{\sin 2x + 2\cos^2 x}$$

$\Rightarrow y^2 - 28y + 27 = 0 \Rightarrow y = 27$ or, $y = 1$

If $y = 27$, then

$3^{\sin 2x + 2\cos^2 x} = 3^3$

$\Rightarrow \sin 2x + 2\cos^2 x = 3$

$\Rightarrow \sin 2x + 2\cos 2x = 2$

$\Rightarrow \sin 2x = 2\cos 2x = 1$

$\Rightarrow \sin x = 0$ or, $\tan x = \dfrac{1}{2}$

If $y = 1$

$\Rightarrow 3^{\sin 2x + 2\cos^2 x} = 1$

$\Rightarrow \sin 2x + 2\cos^2 x = 0$

$\Rightarrow 2\cos x(\sin x + \cos x) = 0$

$\Rightarrow \cos x = 0$ or, $\tan x = -1$

813 **(c)**

Let $f(x) = 27^{\cos 2x} 81^{\sin 2x} = 3^{3\cos 2x + 4\sin 2x}$

$= 3^{5\left(\frac{3}{5}\cos 2x + \frac{4}{5}\sin 2x\right)}$

Let $\frac{3}{5} = \sin \phi$

$\Rightarrow \dfrac{4}{5} = \cos \phi$

Then, $f(x) = 3^{5(\sin \phi \cos 2x + cos\phi \sin 2x)}$

$= 3^{5(\sin(\phi + 2x))}$

For minimum value of given function,

$\sin(\phi + 2x)$ will be minimum

ie, $\sin(\phi + 2x) = -1$

$\therefore \quad f(x) = 3^{5(-1)} = \dfrac{1}{243}$

814 **(c)**

We have,

$\sec 2x - \tan 2x = \dfrac{1 - \sin 2x}{\cos 2x}$

$\Rightarrow \sec 2x - \tan 2x = \dfrac{1 - \cos\left(\frac{\pi}{2} - 2x\right)}{\sin\left(\frac{\pi}{2} - 2x\right)}$

$\Rightarrow \sec 2x - \tan 2x = \dfrac{2\sin^2(\pi/4 - x)}{2\sin(\pi/4 - x)\cos(\pi/4 - x)} = \tan\left(\frac{\pi}{4} - x\right)$

815 **(a)**

$\because \sin^5 x - \cos^5 x = \dfrac{\sin x - \cos x}{\sin x \cos x}$

$\Rightarrow \sin x \cos x \left[\dfrac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$

$\Rightarrow \dfrac{1}{2}\sin 2x[\sin^4 x + \sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^2 x + \cos^4 x] = 1$

$\Rightarrow \sin 2x[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$+ \sin x \cos x(\sin^2 x + \cos^2 x) + \sin^2 x + \cos^2 x] = 2$

$\Rightarrow \sin 2x[1-\sin^2 x \cos^2 x+\sin x \cos x]=2$
$\Rightarrow \sin^3 2x-2\sin^2 2x-4\sin 2x+8=0$
$\Rightarrow (\sin 2x-2)^2(\sin 2x+2)=0$
$\Rightarrow \sin 2x = \pm 2$, which is not possible for any x

816 **(b)**

$\cos(\alpha+\beta)=\frac{4}{5} \Rightarrow \alpha+\beta \in$ 1st quadrant and

$\sin(\alpha-\beta)=\frac{5}{13}$

$\Rightarrow \alpha-\beta \in$ 1st quadrant

$\Rightarrow 2\alpha=(\alpha+\beta)+(\alpha-\beta)$

$\therefore \tan 2\alpha=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$

$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}}=\frac{56}{33}$

817 **(d)**

We have,

$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$

$=\frac{1}{2\sin\frac{\pi}{7}}\left\{2\sin\frac{\pi}{7}\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}\right\}$

$=\frac{1}{2\sin\frac{\pi}{7}}\left\{\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\pi-\sin\frac{5\pi}{7}\right\}$

$=-\frac{1}{2}$

818 **(a)**

We have,

$2a\cos^2\left(\frac{C}{2}\right)+2c\cos^2\left(\frac{A}{2}\right)=3b$

$\Rightarrow a(1+\cos C)+c(1+\cos A)=3b$

$\Rightarrow a+c+(a\cos C+c\cos A)=3b$

$\Rightarrow a+c+b=3b \Rightarrow a+c=2b \Rightarrow a,b,c$ are in A.P.

819 **(d)**

Given that, $\frac{1-\cos 2\theta}{1+\cos 2\theta}=3$

$\Rightarrow \frac{2\sin^2\theta}{2\cos^2\theta}=3$

$\Rightarrow \tan^2\theta=(\sqrt{3})^2$

$\Rightarrow \tan^2\theta=\tan^2\frac{\pi}{3}$

$\Rightarrow \theta=n\pi\pm\frac{\pi}{3}$

820 **(b)**

We have,

$a = 5$ cm, $b = 4$ cm and $\cos(A-B) = \frac{31}{32}$

$$\therefore \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\Rightarrow \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \frac{a-b}{a+b}\sqrt{\frac{1+\cos C}{1-\cos C}}$$

$$\Rightarrow \frac{1-\frac{31}{32}}{1+\frac{31}{32}} = \left(\frac{5-4}{5+4}\right)^2\left(\frac{1+\cos C}{1-\cos C}\right)$$

$$\Rightarrow \frac{81}{63} = \frac{1+\cos C}{1-\cos C} \Rightarrow \cos C = \frac{1}{8}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	B	A	D	C	A	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	C	C	A	B	D	A	D	B